TM241498L

Reg. No :..............................

Name :..................................

MASTER'S DEGREE (C.S.S) EXAMINATION, NOVEMBER 2024

2024 ADMISSIONS REGULAR

SEMESTER I - CORE COURSE

ST1C03TM - Analytical Tools for Statistics

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Define vector space and subspace.
2. Explain sum of subspaces.
3. Define inner product.
4. Define row rank and column rank of a matrix.
5. Explain system of linear equations.
6. Explain eigen values and eigen spaces.
7. Explain spectral representation of a real symmetric matrix.
8. Show that every positive definite matrix or positive semi definite matrix can be represented as a Gram-matrix.
9. Explain indefinite quadratic form.
10. Explain trace of a matrix.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Let S and T be two subspaces of vector space V. Then show that $S \cap T$ and $S + T$ are subspaces of V.
12. State and prove the necessary and sufficient conditions for a non-empty set W to be a subspace of a vector space V.
13. Find the generalized inverse of the matrix A = $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \\ 2 & 4 & 6 & 8 \end{bmatrix}$.
14. Show that a g-inverse always exist and it is not unique.
15. If A = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ then show that $6(2I - A)^{-1} = A^2 + 2A + 3I$.
16. Prove that the Geometric multiplicity of a characteristic root cannot exceed algebraic multiplicity of the same.
17. Show that every quadratic form can be reduced to a form containing square terms only by a nonsingular linear transformation.
18. If A is a real symmetric matrix, the show that |A| is the product of characteristic root and trace (A) is the sum of characteristic roots.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. a) Define the basis and dimension of a vector space and also give examples for both.

    b) If W_1 and W_2 are finite dimensional subspaces of a vector space V, then show that $d(W_1 + W_2) = d(W_1) + d(W_2) - d(W_1 \cap W_2)$

20. (a) Show that the rank of product of two matrices cannot exceed the rank of either matrices.

(b) If A and B are two square matrices of the same order n then, $r(AB) \geq r(A) + r(B) - n.$

21. Prove that any set of characteristic vectors x1,x2,x3,.....,xk corresponding respectively to a set of distinct characteristic roots of a matrix is linearly independent.

22. Examine the definiteness of the quadratic form $6x^2 + 3y^2 + 14z^2 + 4yz + 18xz + 4xy$ after reducing it to its canonical form.