TM241592F

Reg. No :..............................

Name :..................................

MASTER'S DEGREE (C.S.S) EXAMINATION, NOVEMBER 2024

2024 ADMISSIONS REGULAR

SEMESTER I - CORE COURSE

ST1C02TM - Distribution Theory

Time : 3 Hours

Maximum Weight : 30

## Part A

**I. Answer any Eight questions. Each question carries 1 weight** **(8x1=8)**

1. Define logarithmic distribution and generalized power series distribution.
2. Define negative binomial distribution and find its p.g.f.
3. Define logistic distribution.
4. Check whether Poisson family belongs to the one parameter exponential family.
5. If X and Y are independent standard normal variate write down the p.d.f of (i) $U = \frac{X}{Y}$ (ii) $V = \frac{X^2}{Y^2}$
6. If X has a $\beta_I(\alpha,\beta)$ distribution, then show that 1-X has $\beta_I$ distribution.
7. Let X and Y be jointly distributed with p.d.f $f(x,y) = \begin{cases} 2; 0 < x < y < 1 \\ 0; otherwise \end{cases}$.
   Find the p.d.f $f(x/y)$ and $f(y/x)$ also find $p[(y > 12)/(x = 12)]$.
8. Find the mode of Chi-square distribution.
9. Define chi-square distribution and F distribution.
10. Show that the two parameter Gamma distribution belongs to the Exponential family of distributions

## Part B

**II. Answer any Six questions. Each question carries 2 weight** **(6x2=12)**

11. Express binomial and logarithmic series distribution as a particular case of power series distribution.
12. State and prove additive property of binomial distribution.
13. Express Poisson as a limiting form of negative binomial distribution.
14. Define Pareto Distribution and mention its important characteristics.
15. If $X$ has Cauchy distribution with p.d.f $f(x) = \frac{1}{\pi(1+x^2)}, -\infty < x < \infty$, show that the distribution of $\frac{1}{X}$ is again Cauchy. Is the converse true? Establish your claim.
16. Let $x$ & $y$ be i.i.d R.V's with j.d.f $f(x,y) = 4xy\, e^{(x^2+y^2)}$ ; $0 < x < \infty; 0 < y < \infty$ Find the density function of $u = \sqrt{x^2+y^2}$
17. Define mean and mode of t-distribution with n d.f.
18. Let $Y = \frac{X_1 - X_2}{2}$ where $X_1$ *and* $X_2$ are i.i.d R.Vs each having $\chi_2^2$. Find the p.d.f of $Y$

## Part C

**III. Answer any Two questions. Each question carries 5 weight** **(2x5=10)**

19. (a) Sate and prove Lack of Memory property

(b)Fit a geometric distribution to the following data

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20. Let X ~ N(0, 1) and Y ~ N(0, 1) be independent random variables. Obtain the distribution of $\frac{|X|}{|Y|}$

21. Give the derivation $\chi^2$ distribution both by the method of moment generating function and method of Inducton.

22. (a) Find the p.d.f of $X_{(r)}$, the rth order statistic in a random sample of size n from the exponential distribution $f(x) = \alpha e^{-\alpha x}, X \geq 0.$

(b) Show that $X_{(r)}$ and $X_{(s)}-X_{(r)}$ ,r<s are independently distributed.