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TM241287V

Reg. No :..............................

Name :..............................

MASTER'S DEGREE (C.S.S) EXAMINATION, NOVEMBER 2024

2020, 2021, 2022, 2023 ADMISSIONS SUPPLEMENTARY

SEMESTER I - CORE COURSE Applied Statistics and Data Analytics

ST1C01TM - Probability and Measure Theory

Time : 3 Hours

Maximum Weight : 30

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Prove that every Borel set is measurable.
2. Prove that outer measure of a singleton set is zero.
3. Define Counting measure
4. Define independence of a sequence of random variables.
5. State continuity property of probability measure
6. State Tchebychev's inequality
7. State Holder's Inequality.
8. State Jensen's inequality.
9. state Liaponov's Central limit theorem
10. State Lindberg-Feller central limit theorem.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. State and prove Fatou's lemma.
12. Define a measurable function and explain its properties.
13. Prove that the set of discontinuity points of distribution function is atmost countable.
14. Differentiate between discrete and continuous probability spaces, and outline the key characteristics associated with each.
15. Define convergence is probability and convergence in distribution regarding a sequence of random variables {Xn, n ≥ 1} to a random variable X. Show that the former implies the latter
16. State and prove Basic inequality.
17. Show that Liaponov's condition implies Lindberg-Feller condition for CLT.
18. Let sequence $\{X_n\}$ be a sequence of independent random variables with $P[X_k = \pm k] = \frac{k^{-\lambda}}{2}, P(X_k = 0) = 1 - k^{-\lambda}$. Identify the range of values of λ for which $\{X_n\}$ holds CLT.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19.

1. State and prove Lebesgue Monotone Convergence theorem.
2. Prove that μ^* is an outer measure.

20. Derive Jordan decomposition theorem.

21. (a) Show how Chebychev's inequality be obtained from Markov's inequality. Also, show that values of Karl Pearson's correlation coefficient lies between -1 and 1.

(b) Show that convergence in probability implies that convergence in distribution under conditions to be stated, and give an example. Establish the converse of this result

22.

a. State and prove Chebychev's WLLN.

b. State and prove Khinchine WLLN.