TB165385E

Reg. No.: ....

Name : ....

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2018
(2016 Admission Regular & 2015 Admission Supplementary)
SEMESTER V- CORE COURSE (MATHEMATICS)
MT5B08B - GRAPH THEORY

Time: Three Hours **Maximum Marks: 80**

PART A

I. Answer all the questions. Each question carries 1 mark.

1. Define a graph.
2. Draw a 2-regular graph.
3. Draw a graph with three connected components.
4. If G is a connected graph with 17 edges what is the maximum possible number of vertices in G?
5. State Dirac's Theorem.
6. Define a directed Hamiltonian cycle.

(6 x 1 = 6)

PART B

II Answer any seven questions. Each question carries 2 marks.

7. Let G be graph with n vertices and e edges and let m be the smallest positive integer such that $\geq \frac{2e}{n}$. Prove that G has a vertex of degree at least m.
8. Prove that it is impossible to have a group of eleven people in a conference where each person knows exactly 3 of others.
9. In any graph G, show that the number of vertices with odd degree is even.
10. Let G be a connected graph. Then prove that G is a tree if and only if every edge of G is a bridge.
11. Prove that an edge e of a graph G is a bridge if and only if e is not part of any cycle in G.
12. Prove that a simple graph is Hamiltonian if and only if its closure is Hamiltonian.
13. Write a short note on Chinese Postman problem.
14. State and prove the first theorem of digraph theory.
15. Define an Euler digraph.
16. Explain Teleprinter's problem

(7 x 2 = 14)

PART C

III Answer any five questions. Each question carries 6 marks.

17. Show that the k-cube graph is bipartite.
18. Show that a tree with n vertices has precisely n-1 edges.
19. Prove that a connected graph G is Euler if and only if it has at most two odd vertices.
20. Prove that a 2- regular graph G has a perfect matching if and only if each component of

G is an even cycle.

21. Let G be a k-regular bipartite graph with $k > 0$. Then show that G has a perfect matching.
22. State and prove Redei's Theorem.
23. Let v be any vertex having maximum out degree in the tournament T. Prove that for every vertex w of T there is a directed path from v to w of length at most 2.
24. Prove that an Euler digraph is strongly connected.

(5 x 6 = 30)

PART D

IV Answer any two questions. Each question carries 15 marks.

25. State and prove the necessary and sufficient condition for a graph G to be bipartite.
26. Let G be a graph with n vertices and q edges. Then show that G has at least $n-\omega(G)$ edges.
27. a) Write a short note on Optimal Assignment Problem.
    b) Write a short note on Personnel Assignment Problem.
    c) Write a short note on Travelling Salesman Problem.
28. Let D be a weakly connected digraph with at least one arc. Then show that D is Euler if and only if $od(v) = id(v)$ for every vertex v of D.

(2 x 15 = 30)