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## B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2018

(2017 Admissions Regular, 2016 Admissions Supplementary/Improvement \& 2015 Admissions Supplementary)
SEMESTER III - COMPLEMENTARY COURSE (MATHEMATICS) MT3CPC03B - VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND ANALYTICAL GEOMETRY
(For Physics and Chemistry)
Time: Three Hours

Maximum Marks: 80

## PART A

I Answer all questions. Each question carries 1 mark

1. Write the vector function representing Helix.
2. Show that $F=(2 x-3) i-2 j+(\cos z) k$ is not conservative.
3. Find the general integrating factor of $x \frac{d y}{d x}+3 y=x^{3}$
4. Write the Bernoulli's equation.
5. Find the polar equation of $x-y=3$
6. What is the eccentricity of the hyperbola $\frac{x^{2}}{9}-\frac{y^{2}}{4}=1$

## PART B

II Answer any seven questions. Each question carries 2 marks
7. Find $\mathbf{N}$ for the curve $\mathbf{r}(\mathrm{t})=(\cos t+t \sin t) \mathbf{i}+(\operatorname{sint}-t \cos t) \mathbf{j}>0$.
8. Find the derivative of $f(x, y)=x e^{y}+\cos (x y)$ at the point $(2,0)$ in the direction of $a=3 i-4 j$.
9. Evaluate $\int_{C}(x-y+z-2)$ ds where $C$ is the straight line segment from $x=t, y=1-t, z=1$, from $(0,1,0)$ to $(1,0,0)$.
10. Find the work done by $\mathbf{F}$ over the curve in the direction of increasing $t$ where $\mathbf{F}=2 \mathrm{y} \mathbf{i}+3 \mathrm{x} \mathbf{j}+(\mathrm{x}+\mathrm{y}) \mathbf{k}, \mathbf{r}(\mathrm{t})=(\cos \mathrm{t}) \mathbf{i}+(\sin \mathrm{t}) \mathbf{j}-\left(\frac{\mathrm{t}}{6}\right) \mathbf{k}, 0 \leq \mathrm{t} \leq 2 \pi$.
11. Find the outward flux of the field $\mathbf{F}(x, y)=x i+y^{2} \mathbf{j}$ across the square bounded by the lines $x$ $= \pm 1$ and $y= \pm 1$.
12. Solve $(x+\sin y) d x+\left(y^{2}+x \cos y\right) d y=0$
13. Solve $y-p x=\frac{p}{1+p}$
14. Find the centre, vertices of the hyperbola $x^{2}-y^{2}-2 x+4 y=4$
15. The parabola $x^{2}=-4 y$ is shifted left 1 unit and 3 unit to generate the parabola $(x+1)^{2}=$ $-4(y-3)$. Find the vertex, foci and directrix of new parabola.
16. Write the parametric representation of the hyperbola $x^{2}-y^{2}=1$

## PART C

## IV Answer any five questions. Each question carries $\mathbf{6}$ marks

17. The velocity of a particle moving in space is given by $\frac{d r}{d t}=\cos \mathrm{t} \mathbf{i}-\sin \mathrm{t} \mathbf{j}+\mathbf{k}$. Find the particles position as a function of $t$ if $\mathbf{r}=2 \mathbf{i}+\mathbf{k w h e n t}=0$.
18. Find the curvature for the curve $r(t)=e^{t} \cos t i+e^{t} \sin t j+2 k$.
19. Find the area of the surface cut from the paraboloid $x^{2}+y^{2}-z=0$ by the plane $z=2$.
20. Use divergence theorem to find the outward flux $F$ across the boundary of the region $D$, where $\mathbf{F}=(\mathrm{y}-\mathrm{x}) \mathbf{i}+(\mathrm{z}-\mathrm{y}) \mathbf{j}+(\mathrm{y}-\mathrm{x}) \mathbf{k}$ and D is the cube bounded by the planes $\mathrm{x}= \pm 1, \mathrm{y}$ $= \pm 1$ and $\mathrm{z}= \pm 1$.
21. Solve $x \frac{d y}{d x}-2 y=\frac{3 y^{4}}{x}, y(1)=1 / 2$
22. Solve $(x+y) d y+(x-y) d x=0$
23. Solve $x\left(\frac{d y}{d x}\right)^{3}-12 \frac{d y}{d x}-8=0$
24. Convert the following equation to polar equation
a) $(x-2)^{2}+y^{2}=4$
b) $x^{2}-y^{2}=1$

## PART D

IV Answer any two questions. Each question carries 15 marks
25. Find $T N$ and $\kappa$ for the space curve $r(t)=(3 \sin t) i+(3 \cos t) j+4 t k$
26. Use stoke's theorem to calculate the circulation of the field $F=2 y i+3 x j-z^{2} k$ around the curve $C=x^{2}+y^{2}=9$ in the $x y$ plane counter clock wise.
27. Sketch the ellipse which include the directrix that corresponds to the focus at the origin
(a) $\mathrm{r}=\frac{25}{10-5 \cos \theta}$
(b) $\mathrm{r}=\frac{400}{16+8 \sin \theta}$
28. Solve the differential equation
a) $y^{2}-1-p^{2}=0$
b) $\frac{x}{y}(\ln x-\ln y-1) d y=-d x, y(1)=e$
(2x15=30)

