15.9

TM244931I

Reg. No :...............................

Name :..................................

MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2024

2022 ADMISSIONS REGULAR

SEMESTER IV - Mathematics

MT4E01TM20 - Multivariate Calculus and Integral Transforms

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Let R = $(-\infty,\infty)$. Assume that f ϵ $L^2(R)$ and g ϵ $L^2(R)$. Then prove that the convolution integral exists for each x in R and the function h is bounded on R.
2. Define Fourier Transforms. Derive the inversion formula for Fourier Transforms.
3. Find the Laplace Transform of the given functions

   i) e^{at} ii) cos at iii)sin at iv) $t^n e^{at}$

4. Derive the Jacobian matrix of a function **f**:$R^n \rightarrow R^m$ at a point **c** with total derivative **T**.
5. Let **f** be a function with values in R^m which is differentiable at each point of **c** in R^n with total derivative **T**. Define m(T), Jacobian matrix of **f** at **c.** Also represent the jacobian matrix in terms of gradient vector.
6. Compute the gradient vectors for the function

$$f(x,y) = \begin{cases} xy\sin\frac{1}{x^2+y^2}, & if(x,y) \neq (0,0) \\ 0 & , if(x,y) = (0,0) \end{cases}$$

7. Verify the mixed partial derivatives $D_{1,2}f$ and $D_{2,1}f$ are equal for the function f(x,y) = tan (x^2/y) ; y $\neq$ 0
8. Let f be a complex valued defined for each complex $z\neq0$ by the equation f(z)=1/z . Show that $J_f(z)=-|(z)|^{-4}$
9. Define k- cell. Derive integral of f over a k- cell.
10. Define k-form of an open set E in R^n .

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. State and prove Weierstrass Approximation theorem.
12. Show that

$$\frac{2}{\pi}\int_0^\infty \frac{\sin v\cos vx}{v}\,dv = \begin{cases} 1 & if -1 < x < 1 \\ 0 & if |x| > 1 \\ 1/2 & if |x| = 1 \end{cases}.$$

13. Show that if f is differentiable at c, then f is continuous at c.
14. Let u and v be two real valued functions defined on a subset S of the complex plane. Assume that u and v are differentiable at an interior point c of S and the partial derivatives satisfy the Cauchy-Riemann Equations at c. Prove that f'(c) = $D_1u(c)$+i $D_1v(c)$.
15. Let f: $S\rightarrow R^m$ be differentiable at an each point of S where S is an open connected subset of R^n . If f'(c) = 0, for each c in S. Prove that f is a constant on S.
16. Let A be an open subset of R^n and assume that f: $A\rightarrow R^n$ is continuous and has finite partial derivatives D_jf_i on A. If $J_f(x)\neq0$ for each x in A, prove that f(A) is open.
17. For every f $\in\zeta$ (I^k), Prove that L(f) = L' (f).

18. a) If ω and λ be k- and m- forms respectively of class ζ' in E, prove that $\mathbf{d(\omega \wedge \lambda) = (d\omega) \wedge \lambda + (-1)^k \omega \wedge d\lambda}$

b) If ω is of class ζ'' in E, prove that $\mathbf{d^2\omega = 0}$.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. a) Define convolution of two Lebesgue integrable function f and g on (-∞,∞).Prove that convolution is commutative. b) State and Prove Convolution Theorem for Fourier Transforms.

20. Let f and D_2F be continuous on a rectangle [a,b]x[c,d]. Let p and q be differentiable on [c,d] where p(y)ϵ [a,b] and q(y) ϵ [a,b] for each y in [c,d]. Define F by the equation $F(y) = \int_{p(y))}^{q(y))} f(x, y)dx$ if yϵ [c,d]. Show that *F'(y)* exist for each y in (c,d) and is given by

$$F'(y) = \int_{p(y))}^{q(y))} D_2 f(x, y)dx + f(q(y), y)q'(y) - f(p(y), y)p'(y)$$

21. a) Define mixed partial derivatives. Show that $D_{r,k}f$ is not necessarily same as $D_{k,r}f$ for every function f

b) If both partial derivatives D_rf andD_kf exist on an n-ball B(c) and if both $D_{r,k}f$ and$D_{k,r}f$ are continuous at c then, show that $D_{r,k}f(c)=D_{k,r}f(c)$.

22. a) Suppose T is a ζ' - mapping of an open set E in R^n into an open set V in R^m , S is a ζ' - mapping of V into an open set W in R^p , and ω is a k-form in W, so that ω_S is a k-form in V and both $(\omega_S)_T$ and ω_{ST} are k-forms in E, where ST is defined by (ST)(x) = S(T(x)).Then prove that $(\omega_S)_T = \omega_{ST}$.

b) Suppose T is a ζ' - mapping of an open set E in R^n into an open set V in R^m , ∅ is a k- surface in E, and ω is a k- form in V. Prove that $\int_{T\phi} \omega = \int_{\phi} \omega_T$