TM244842S

Reg. No :..............................

Name :..................................

MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2024

2022 ADMISSIONS REGULAR

SEMESTER IV - MATHEMATICS

MT4E03TM20 - Analytic Number Theory

Time : 3 Hours

Maximum Weight : 30

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Prove that d(n) is odd if and only if n is a square.
2. Show that [2x]-2[x] is either 0 or 1.
3. Find all integers n such that φ(n)=φ(2n).
4. Write a short note on Chebishev's functions.
5. State Abel's Identity and deduce Euler Summation formula from it.
6. Find the solutions of the quadratic congruence $x^2 \equiv 1 \pmod 8$.
7. Show that if $a \equiv b \pmod m$ then $a^n \equiv b^n \pmod m$.
8. Find the remainder when 41^{75} is divided by 3.
9. Prove that the number of partitions of n into m parts is equal to the number of partitions of n into parts, the largest of which is m.
10. Prove or disprove : p(5) is a multiple of 7

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Prove that if f and g are multiplicative so is their Dirichlet product.
12. Prove that $\sum_{n\leq x} n^{\alpha} = \frac{x^{\alpha+1}}{\alpha+1} + O(x^{\alpha}) if \alpha \geq 0$
13. Show that $\lim_{x\to\infty}\left(\frac{\psi(x)}{x} - \frac{\vartheta(x)}{x}\right) = 0$.
14. Let $a_1<a_2<$ ... $<a_n\leq x$ be a set of positive integers such that no a_i divides the product of the others then prove that $n\leq\pi(x)$.
15. State and prove Wilson's Theorem.
16. For a given modulus m show that the m residue classes $\hat{1}, \hat{2}, \ldots, \hat{m}$ are disjoint and their union is the set of all integers.
17. Prove that m is prime if and only if exponent of a modulo m = m − 1 for some a
18. Let p be an odd prime and let d be any positive divisor of p - 1. Then prove that in every reduced residue system mod p there are exactly $\varphi(d)$ numbers a such that $exp_p(a) = d$ and in particular, when $d = \varphi(p) = p - 1$ there are exactly $\varphi(p-1)$ primitive roots mod p.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. a) Derive Dirichlet's formula for the partials sums of the divisor function d(n).

    b) Show that the set of lattice points visible from the origin has density $6/\pi^2$.

20. For every integer n ≥ 2, prove that $\frac{1}{6}\frac{n}{\log n} < \pi(n) < 6\frac{n}{\log n}$.

21. State and prove Wolstenholme's theorem.

22. (i) Define Pentagonal Numbers.

(ii) If |x|<1, Show that $\prod_{m=1}^{\infty}(1-x)^m = 1 - x + x^2 - \cdots$