TM244187J

Reg. No :.....

Name :.....

MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2024

2022 ADMISSIONS REGULAR

SEMESTER IV - M. Sc. Mathematics ELECTIVE COURSE

MT4E02TM20 - Combinatorics

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Explain (i) Injection Principle. (ii) Bijection Principle. (iii) Bell's number.
2. Find the number of ways of arranging the 26 letters in the English alphabet in a 5 letters between x and y.
3. Using Pigeonhole Principle prove that any group of 13 students in a class must h in the same month
4. Let $A = \{a_1, a_2, \cdots, a_5\}$ be a set of positive integers. Show that for any permut the product $(a_{i1} - a_1) \cdot (a_{i2} - a_2) \cdots (a_{i5} - a_5)$ is always even.
5. Show that $D_n = (n-1)(D_{n-1} + D_{n-2})$.
6. Show that $\lim_{n\to\infty} \frac{D_n}{n!} = e^{-1}$
7. Let $S = \{1, 2, \cdots 500\}$. Find the number of integers in S which are divisible by 3, 5
8. Define recurrence relation for any sequence $a(n)$, solution of a recurrence relation a
9. Define r^{th} order linear homogenous recurrence relation for any sequence $a(n)$
10. List the partitions of 6 and hence find $p(6)$.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Show that for any finite X, the cardinality of X and cardinality of it's power set ar
12. Let $S = \{3001, 3002, \cdots 4999\}$. Find the odd integers in S in which no digit is repe
13. Among any 5 points in an equilateral triangle of unit lengths, show that there are t
14. Find the relation between $R(p, q)$ and $R(p-1, q) + R(p, q-1)$, where $p, q \geq 2$.
15. Find the number of prime numbers which do not exceed 120 using Generalized Prin
16. A man has 6 friends. At dinner in certain restaurant, he has met each of them l every three of them 4 times, every four of them 3 times, every five of them twice an He has dined out 8 times without meeting any of them. How many times he has di
17. Find the coefficient of x^9 and x^{14} in the expansion of $(1+x+x^2+\cdots x^5)^4$.
18. Show that the number of partitions of n into distinct parts is equal to number c Verfy the theorem for $n = 4, 5$.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. (i)Find the number of non-negative integer solutions to the equation

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$rx_1 + x_2 + \cdots + x_n = kr$ where $r, k, n \in N$

(*ii*) In a group of 15 students, 5 of them are female. If exactly three female stud in how many ways can 9 students be chosen from the group (a)to form a committe

20. (*a*)Show that $R(p,q) \leq R(p-1,q) + R(p,q-1) - 1$, if $R(p-1,q), R(p,q-1)$ a (*b*)Define Ramsey's number
(*c*)State Ramsey's Theorem and Generalized Pigeon Hole Principle

21. Derive the formula for $D(n,r,k)$, the number of $r-$ permutations of N_n that ha Also derive any two identities satisfied by D_n, the number of derangements of N_n.

22. Solve the recurrence relation $a_n - 3a_{n-1} + 2a_{n-2} = 2^n, a_0 = 3, a_1 = 8$.

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