11.4

TM244114U

Reg. No :.....

Name :.....

MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2024

2022 ADMISSIONS REGULAR

SEMESTER IV - CORE COURSE MATHEMATICS

MT4C17TM20 - Optimization Techniques

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Explain general Integer Linear Programming and Mixed Integer Linear Programming problems.
2. Explain the terms pruned and fathomed associated with branch and bound method.
3. Define (a) Partial Graph (b) Sub graph.
4. Prove that the maximum flow in a graph is equal to the minimum of the capacities of all possible cuts in it.
5. Explain a cut in the context of maximum flow problem.
6. Explain (i) Zero Sum Two Person game (ii) Pay off
7. Examine the following pay off matrix for saddle points.
$$\begin{bmatrix} 2 & -1 & -2 \\ 1 & 0 & 1 \\ -2 & -1 & 2 \end{bmatrix}$$
8. What are the necessary and sufficient conditions for the local minimum of an unconstrained function in one variable?
9. Calculate the value of $f(x) = 2x^3 - 3x^2 + x - 4$ at $x = 7$ starting from the point $x_0 = 3$.
10. Explain the procedure for Golden section search.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Maximize $2x_1 + 5x_2$ Subject to $0 \le x_1 \le 8, 0 \le x_2 \le 8$ and either $4 - x_1 \ge 0$ or $4 - x_2 \ge 0$.
12. Solve by Branch and Bound method.
Maximize $z = 2x_1 + 6x_2$
subject to $3x_1 + x_2 \le 5, 4x_1 + 4x_2 \le 9$, x_1, x_2 non negative integers.
13. Explain the concept of Duality in Maximum Flow Problem.
14. Find the minimum path from v_1 to v_8

Arc	(1,2)	(1,3)	(1,4)	(2,3)	(2,6)	(2,5)	(3,5)	(3,4)	(4,7)
Length	1	4	11	2	8	7	3	7	3
Arc	(5,6)	(5,8)	(6,3)	(6,4)	(6,7)	(6,8)	(7,3)	(7,8)	
Length	1	12	4	2	6	10	2	2	

15. Applying the notion of dominance simplify the following pay off matrix.
$$\begin{bmatrix} 4 & -8 & 7 & -2 \\ 3 & -9 & 2 & -3 \\ -2 & 6 & 8 & 2 \end{bmatrix}$$

16. Find the saddle point, if any, of the following game. Find the optimal strategies and value of the game.

$$\begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix}$$

17. Solve using Newton's method. Minimize $2(x_1 + x_2)^2 + 2x_1^2 + 2x_2^2$ with initial point (5,2).

18. Maximize $f(x) = 3x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2$ subject to $2x_1 - x_2 = 4$ using Lagrange multipliers.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. Solve by cutting plane method : Maximize $3x_1 + 4x_2$ subject to $2x_1 + 4x_2 \leq 13$, $-2x_1 + x_2 \leq 2$, $2x_1 + 2x_2 \geq 1$, $6x_1 - 4x_2 \leq 15$, $x_1, x_2 \geq 0$, x_1, x_2 integers.

20. Find the maximum non negative flow in the network described below, arc (v_j, v_k) being denoted as (j,k), v_a is the source and v_b the sink.

Arc	(a,1)	(a,2)	(1,2)	(1,3)	(1,4)	(2,4)	(3,2)	(3,4)	(4,3)	(3,b)	(4,b)
Capacity	8	10	3	4	2	8	3	4	2	10	9

21. Solve graphically the game whose pay off matrix is.

$$\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix}$$

22. Maximize the function $f(X) = -3X^2 + 21.6X + 1.0$ with a minimum resolution of 0.50 over 6 functional evaluations using Fibonacci search technique. The optimal value of f(X) is assumed to lie in the range $25 \geq X \geq 0$.