TM242975J

Reg. No :...............................

Name :.................................

MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2024

2023 ADMISSIONS REGULAR

SEMESTER II - CORE COURSE

ST2C08TM - Multivariate Distributions

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Give the properties of Marshall - Olkin bivariate exponential distribution.
2. Define second form of Gumbel's bivariate exponential distribution.
3. Define first form of Gumbel's bivariate exponential distribution.
4. Distinguish between simple correlation and partial correlation.
5. Derive the distribution of the sample mean vector $\overline{X}$ if $X \sim N_p(\mu, \Sigma)$.
6. Let $C = [C_{\alpha\beta}]$ be an orthogonal matrix and $Y_\alpha = \sum_{\beta=1}^{N} C_{\alpha\beta} X_\beta$, then show that $\sum_{\alpha=1}^{N} Y_\alpha Y_\alpha' = \sum_{\alpha=1}^{N} X_\alpha X_\alpha'$
7. Explain the derivative of a scalar function with respect to a vector.
8. Explain the Jacobian of linear matrix transformation.
9. State Cochran's theorem.
10. Let $Y'Y = Q_1 + Q_2$ where $Q_1 \sim \chi^2_{(a)}$ then show that, $Q_2 \sim \chi^2_{(n-a)}$, where n is the order of the matrix A.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Given (X, Y) is a Bivariate normal distribution. Show that X+Y and X-Y are independent if and only if $\sigma_x^2 = \sigma_y^2$.
12. For Gumbel's first form of Bivariate Exponential distribution find the conditional distribution f(y/x).
13. If $X \sim N_p(\mu, \Sigma)$, then X can be written as $X = \mu + CY$ where, $\Sigma = C'C''$ and $Y \sim N_p(0, I)$.
14. State and prove the reproductive property of Mutivariate normal distribution.
15. Let A and Σ be partitioned in to $P_1, P_2, \ldots\ldots, P_q$ rows and columns $P_1 + P_2 + \ldots\ldots + P_q = P$.

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1q} \\ A_{21} & A_{22} & \cdots & A_{2q} \\ \cdots & \cdots & \cdots & \cdots \\ A_{q1} & A_{q2} & \cdots & A_{qq} \end{pmatrix}, \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} & \cdots & \Sigma_{1q} \\ \Sigma_{21} & \Sigma_{22} & \cdots & \Sigma_{2q} \\ \cdots & \cdots & \cdots & \cdots \\ \Sigma_{q1} & \Sigma_{q2} & \cdots & \Sigma_{qq} \end{pmatrix}$$

If $\sum_{ij} = 0 \, for \, i \neq j$ and if $A \sim W(\Sigma, n)$ then,

Show that $(A_{jj})'s$ are independently distributed as $W(\Sigma_{jj}, n)$.

16. Let X be a real symmetric pxp matrix and T is a lower triangular matrix with $t_{jj} > 0, j = 1, 2, \ldots$ then if X = TT' Show that

$$dX = 2^p \prod_{j=1}^{p} (t_{jj})^{p-j+1} dT.$$

17. Show that if $Y \sim N(0, I)$, then a necessary and sufficient condition for the Quadratic form $Y'AY$ to be distributed as a χ^2 is that A is an idem potent matrix.

In this case the degrees of freedom of χ^2 = rank (A) = Trace (A).

18. Write a short note on the distribution of sample partial correlation coefficient based on N observations.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. Derive the Moment generating function of bivariate normal distribution.
20. Derive the Probability density function of a Multivariate normal distribution.
21. Derive the p.d.f of Wishart distribution.
22. Derive the sampling distribution of partial correlation coefficient for the null case.