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TM242995Z

Reg. No :.....

Name :.....

MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2024

2023 ADMISSIONS REGULAR

SEMESTER II - CORE COURSE Applied Statistics and Data Analytics

ST2C06TM - Estimation Theory

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Define minimal sufficient statistic and give example of the same.
2. Show that beta distribution belongs to the exponential family.
3. Let X_1, X_2 be two i.i.d P(λ) random variables. Show that the statistic $T = X_1 + 2X_2$ is not sufficient for λ.
4. Obtain the cramer rao lower bound for the variance of an unbiased estimator of θ in sampling from N(θ,1).
5. State the relation between Cramer Rao lower bound and Bhattacharyya's bound.
6. Explain moment method of estimation.
7. Define Modified minimum chi square method of estimation.
8. What are the assumptions of Cramer-Huzurbazar theorem.
9. Define Bayes rule.
10. What is meant by posterior distribution?

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. State and prove Basu's theorem.
12. Let X_1, X_2, X_3.....X_n be i.i.d U(0,θ). Show that $M_n = Max(X_1, X_2, \dots, X_n)$ is complete sufficient for θ.
13. Explain CRLB and its regularity conditions
14. Show that Minimum Variance Bound Estimator is Unique.
15. Show that MLE need not be unique.
16. Find the moment estimate of parameters of (i) negative binomial distribution (ii) Poisson distribution.
17. Explain statistical decision problem.
18. Explain conjugate family with an example.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. State and Prove Rao-Blackwell theroem.

20.
    a. Find the MVB for the parameter in Bernoulli Distribution. Also find its variance.
    b. Find Cramer-Rao lower bound for any unbiased estimator based on n independent observations for θ in $f(x) = \frac{1}{\pi} \frac{1}{1+(x-\theta)^2}$.

21. a. Define Maximum likelihood estimator of a parameter.
    b. Explain the properties of MLE.
    c. Is it exist always? justify.

22. a.)Define Posterior risk. Prove that the Baye's risk can be obtained by minimizing posterior risk.
    b.)Obtain the posterior probability and Bayes estimate for Bernoulli and Binomial population.