TM242589N

Reg. No :..............................

Name :..................................

MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2024

2023 ADMISSIONS REGULAR

SEMESTER II - CORE COURSE

MT2C10TM20 - Partial Differential Equations

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Define Pfaffian differential equation in n variables.
2. Eliminate the arbitrary function *f* and derive the partial differential equation from the equation $z=x+y+f(xy)$
3. Explain the 3 classes of integrals of a non linear Partial differential equation.
4. Find the general solution of the linear partial differential equation $z(xp-yq)=y^2-x^2$.
5. If u is a function f x,yand z satisfying the partial differential equation $(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$. Show that u contains x ,y and z only in combinations of $x+y+z \text{ and } x^2+y^2+z^2$.
6. If $\beta_r D' + \gamma_r$ is a factor of F(D,D') and $\phi_r(\xi)$ is an arbitrary function of the single variable ξ, then prove that for $\beta_r \neq 0$, $u_r = exp\left(-\frac{\gamma_r x}{\beta_r}\right)\phi_r(\beta_r x)$ is a solution of the equation $F(D,D')z=0$.
7. Solve the equation $\frac{\partial^4 z}{\partial x^4}+\frac{\partial^4 z}{\partial y^4}=2\frac{\partial^4 z}{\partial x^2 \partial y^2}$.
8. Derive the condition for any one parameter family of surfaces f(x,y,z) =c to form a family of equipotential surfaces.
9. Define steam function and velocity potential.
10. Show that $\log(x^2+y^2)$ satisfies Laplace equations.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Find the orthogonal trajectories on the sphere $x^2+y^2+z^2=a^2$ of its intersections with the paraboloids $xy=cz$, c being a parameter.
12. Find the integral curves of the equation $\frac{adx}{(b-c)yz}=\frac{bdy}{(c-a)zx}=\frac{cdz}{(a-b)xy}$
13. Find the general integral of the linear partial differential eqaution $(2xy-1)p+(z-2x^2)q=2(x-yz)$. Also find the particular integral which passes throught the line x =1, y = 0.
14. Find the equation of the system of surfaces which cut orthogonally the cones of the system $x^2+y^2+z^2=cxy$.
15. Solve using Jacobi's method $z^2=pqxy$
16. Solve the equation $r-s+2q-z=x^2y^2$
17. Solve the equation $z(qs-pt)=pq^2$.

18. Show that the surfaces $x^2 + y^2 + z^2 = cx^{2/3}$ can form a family of equipotential surfaces and find the corresponding potential function

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. a) Prove that the neccessary and sufficient condition for a Pfaffian differential equation $X.dr = 0$ to be integrable is that $X.curlX = 0$.

b) Verify that the differential equation $z(z+y^2)dx + z(z+x^2)dy - xy(x+y)dz = 0$ is integrable and find the primitive.

20. a) Find the complete integral of the equation $z^2 = pqxy$.

b) Show that the differential equation $2xz + q^2 = x(xp + yq)$has a complete integral $z + a^2x = axy + bx^2$ and deduce that $x(y+hx)^2 = 4(z - kx^2)$ is also a complete integral.

21. a) Prove that $F(D,D')\{e^{ax+by}\phi(x,y)\} = e^{ax+by}F(D+a,D'+b)\phi(x,y)$

b) Show that a linear partial differential equation of the type $\sum_{r,s} c_{rs}x^r y^s \frac{\partial^{r+s}z}{\partial x^r \partial y^s} = f(x,y)$ may be reduced to one with constant coefficients by the substitution $\xi = \log x,\ \eta = \log y$. Hence solve the equation $x^2r - y^2t + xp - yq = \log x$.

22. a) Show that the family of right circular cones $x^2 + y^2 = az^2$ form a set of equipotential surfaces and show that the corresponding potential function is of the form $A \log\left(\tan\frac{\theta}{2}\right) + B$ where A and B are constants.

b) By seperating the variables $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$

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