

MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2024

2023 ADMISSIONS REGULAR

SEMESTER II - CORE COURSE

MT2C10TM20 - Partial Differential Equations

Time : 3 Hours

Maximum Weight : 30

Part A

I. Answer any Eight questions. Each question carries 1 weight

(8x1=8)

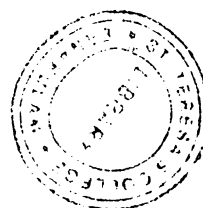
1. Define Pfaffian differential equation in n variables.
2. Eliminate the arbitrary function f and derive the partial differential equation from the equation $z = x + y + f(xy)$
3. Explain the 3 classes of integrals of a non linear Partial differential equation.
4. Find the general solution of the linear partial differential equation $z(xp - yq) = y^2 - x^2$.
5. If u is a function of x, y and z satisfying the partial differential equation $(y-z)\frac{\partial u}{\partial x} + (z-x)\frac{\partial u}{\partial y} + (x-y)\frac{\partial u}{\partial z} = 0$. Show that u contains x, y and z only in combinations of $x+y+z$ and $x^2+y^2+z^2$.
6. If $\beta_r D' + \gamma_r$ is a factor of $F(D, D')$ and $\phi_r(\xi)$ is an arbitrary function of the single variable ξ , then prove that for $\beta_r \neq 0$, $u_r = \exp\left(-\frac{\gamma_r x}{\beta_r}\right) \phi_r(\beta_r x)$ is a solution of the equation $F(D, D')z = 0$.
7. Solve the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2 \frac{\partial^4 z}{\partial x^2 \partial y^2}$.
8. Derive the condition for any one parameter family of surfaces $f(x, y, z) = c$ to form a family of equipotential surfaces.
9. Define stream function and velocity potential.
10. Show that $\log(x^2 + y^2)$ satisfies Laplace equations.

Part B

II. Answer any Six questions. Each question carries 2 weight

(6x2=12)

11. Find the orthogonal trajectories on the sphere $x^2 + y^2 + z^2 = a^2$ of its intersections with the paraboloids $xy = cz$, c being a parameter.
12. Find the integral curves of the equation $\frac{adx}{(b-c)yz} = \frac{bdy}{(c-a)zx} = \frac{cdz}{(a-b)xy}$
13. Find the general integral of the linear partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$. Also find the particular integral which passes through the line $x = 1, y = 0$.
14. Find the equation of the system of surfaces which cut orthogonally the cones of the system $x^2 + y^2 + z^2 = cxy$.
15. Solve using Jacobi's method $z^2 = pqxy$
16. Solve the equation $r - s + 2q - z = x^2 y^2$
17. Solve the equation $z(qs - pt) = pq^2$.



18. Show that the surfaces $x^2 + y^2 + z^2 = cx^{2/3}$ can form a family of equipotential surfaces and find the corresponding potential function

Part C

III. Answer any Two questions. Each question carries 5 weight

(2x5=10)

19. a) Prove that the necessary and sufficient condition for a Pfaffian differential equation $X.dr = 0$ to be integrable is that $X.curlX = 0$.
 b) Verify that the differential equation $z(z + y^2)dx + z(z + x^2)dy - xy(x + y)dz = 0$ is integrable and find the primitive.
20. a) Find the complete integral of the equation $z^2 = pqxy$.
 b) Show that the differential equation $2xz + q^2 = x(xp + yq)$ has a complete integral $z + a^2x = axy + bx^2$ and deduce that $x(y + hx)^2 = 4(z - kx^2)$ is also a complete integral.
21. a) Prove that $F(D, D')\{e^{ax+by}\phi(x, y)\} = e^{ax+by}F(D + a, D' + b)\phi(x, y)$
 b) Show that a linear partial differential equation of the type $\sum_{r,s} c_{rs}x^r y^s \frac{\partial^{r+s} z}{\partial x^r \partial y^s} = f(x, y)$ may be reduced to one with constant coefficients by the substitution $\xi = \log x, \eta = \log y$. Hence solve the equation $x^2 r - y^2 t + xp - yq = \log x$.
22. a) Show that the family of right circular cones $x^2 + y^2 = az^2$ form a set of equipotential surfaces and show that the corresponding potential function is of the form $A \log \left(\tan \frac{\theta}{2} \right) + B$ where A and B are constants.
 b) By separating the variables $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k} \frac{\partial z}{\partial t}$

