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TM242847P

Reg. No :................................

Name :..................................

MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2024

2023 ADMISSIONS REGULAR

SEMESTER II - CORE COURSE MATHEMATICS

MT2C07TM20 - Advanced Topology

Time : 3 Hours

Maximum Weight : 30

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Prove that a compact space in a Hausdorff space is closed.
2. Define (i) Urysohn function. (ii) Extension of a function with example.
3. Define (i) Cube (ii) Hilbert Cube.
4. Define

   i. Product topology on πX_i

   ii. Box

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5. Prove that the Intersection of any family of boxes is a box and also prove that the Intersection of finite number of large boxes is a large box.
6. Prove that, "Sequentially compactness implies Countably Compactness."
7. Define Evaluation function of the indexed family.
8. Let (X,$\mathcal{T}$) be a space. Let D be the indiscrete topology on X. Prove that the function $id_X : X \to X$ is $\mathcal{T}$-D continuous and that the family $\{id_X\}$ distinguishes points of X.
9. Let $S: D \to X$ is a net in a space X and for each $n \in D, A_n = \{S_m: m \in D, m \geq n\}$ prove that a point $x \in X$ is a cluster point of S iff $x \in \bigcap_{n \in D} \overline{A_n}$?
10. Prove that the neighborhood system at point x, where x be any point in the topological space X, will form a directed set.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Prove that, "Every map from a compact space into a T2-space is closed. The range of such a map is a quotient space of the domain."
12. Let A be a subset of a space X and $f : A \to R$ be continous. Then show that any two extensions of f to X agrees on $\bar{A}$
13. Prove that, A topological space is Tychnoff iff each co-ordinate space is so.
14. Prove that if each co-ordinate space is T_3 then the topological product has the corresponding property.
15. State and prove Urysohn Metrisation Theorem.
16. Prove that the Let $\{f_i: X \to Y_i / i \in I\}$ be a family of functions which distinguishes points from closed sets in X then the corresponding evaluation function $e: X \to \prod_{i \in I} Y_i$ is open when regarded as a function from X to e(X).
17. Show that if h, h': X → Y are homotopic and k, k': Y → Z are homotopic, then k ∘ h and k '∘ h' are homotopic.

18. Let X be a topological product of a family of spaces $X_i, i \in I\}$ prove that $S: D \to X$ converges to a point x in X iff for each $i \in I$ the net $\pi_i \circ S$ converges to $\pi_i(x)$ in X_i.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. Show that, if A is a closed subset of a normal space X and suppose $f : A \to [-1, 1]$ is a continous function. Then there exist a continous function $F : X \to [-1, 1]$ such that $F(x) = f(x) \forall x \in A$. Discuss the same if [-1,1] replaced with (-1,1).

20. Prove that, A product of spaces is connected iff each co-ordinate space is connected.

21. State and prove Embedding Lemma.

22. Given spaces X and Y , let [X, Y] denote the set of homotopy classes of maps of X into Y . (i) Let I = [0, 1]. Show that for any X, the set [X, I] has a single element. (ii) Show that if Y is path connected, the set [I, Y] has a single element.

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