8.4

TM242800Q

Reg. No :...............................

Name :.................................

MASTER'S DEGREE (C.S.S) EXAMINATION, MARCH 2024

2023 ADMISSIONS REGULAR

SEMESTER II - CORE COURSE ABSTRACT ALGEBRA

MT2C06TM20 - Abstract Algebra

Time : 3 Hours

Maximum Weight : 30

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Define (i) Factor group/Quotient group. (ii) Conjugation of x by g.
2. Prove that, Let X be a G-Set then prove that G_x is a subgroup of G for each x in X.
3. Prove that the following 3 conditions are equivalent

   i. $ghg^{-1} \in H, \forall g \in H \text{ and } h \in H$

   ii. $gHg^{-1} = H, \forall g \in H$

   iii. $gH = Hg, \forall g \in G.$

4. Let G be a group of order p^n and let X be a finite G-set. Then prove that $|X| \equiv |X_G| (mod\, p)$
5. Prove that no group of order 33 is simple
6. Define (i) join of two subgroups (ii) solvable group
7. i. Define Euler phi-function and find $\phi(12)$.

   ii. Define Mersenne primes and show that $2^{11213} - 1$ is not divisible by 11.
8. If a and m are relatively prime integers then for any integer b prove that ax≡b (mod m) has as solutions all integers in precisely one residue class modulo m.
9. Explain projection homomorphism.
10. If R is a ring with unity and characteristic n, then show that it contains a subring isomorphic to Zn

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Find the number of distinguishable ways the edges of an equilateral triangle can be painted if four different colors of paint are available, assuming only one color is used on each edge and the same color may be used on different edges.
12. Let $H \leq G$. Show that the left coset multiplication $(aH)(bH) = (ab)H$ is well defined if and only if H is a normal subgroup of G.
13. Let N is a normal subgroup of G and if H is any subgroup of G, then H∨N=HN=NH. Furthermore, If H is also normal in G, then prove that HN is normal in G.
14. Let G be a group containing normal subgroups H and K such that H∩K={e} and H∨K=G. then prove that G is isomorphic to H x K.

15.

i. Prove that $f(x) = x^3 + 3x + 2 \ in \ Z_5[x]$ is irreducible over Z_5.

ii. Prove that let $f(x) \in F[x]$ and let f(x) be of degree 2 or 3, then f(x) is irreducible over F if and only if it has a zero in F.

16.

i. State and prove Little Theorem of Fermat.

ii. Find the remainder of 8^{103} when divided by 13.

17. Let R be a ring with unity. Show that an ideal $N \neq R$ is a prime ideal if and only if R/N is an integral domain.

18. (i) Let $G = \{e, a\}$ be a cyclic group of order 2 and $Z_2 = \{0, 1\}$ is a field. Find the group Algebra Z_2G
(ii) Define ring homomorphism and its kernel.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. (i) Prove or disprove: Converse of Lagrange's theorem is true (ii) Show that converse of Lagrange's theorem for abelian group is true.

20.

i. Prove or disprove: There is a group of order 36 is simple

ii. Let H be a subgroup of G and let N be a normal subgroup of G. Then show that $(HN)/N \simeq H/(H \cap N)$

21. (i) Consider two elements f(x) and g(x) in F[x] of degrees n and m respectively. Explain division algorithm for these two elements.

(ii) Show that an element $a \in F$ is a zero of $f(x) \in F[x]$ if and only if $(x - a)$ is a factor of f(x) in F(x).

22. Write a brief note on Quaternions.