

B. Sc DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2018
(2017 Admission Improvement / Supplementary and 2015 & 2016 Admission
Supplementary)
SEMESTER I - COMPLEMENTARY COURSE (MATHEMATICS)
MT1CPC01B – CALCULUS
(Common for Physics and Chemistry)

Time: Three Hours

Maximum Marks: 80

PART A

I. Answer all questions. Each question carries 1 mark

1. Find $\int_0^{\pi} (1 + \cos x) dx$.
2. Let f be an even function such that $\int_0^1 f(x) dx = 3$. Find $\int_{-1}^0 f(x) dx$.
3. Find the length of the curve $x = 1-t, y = 2+3t, -\frac{2}{3} \leq t \leq 1$
4. If $\lim_{x \rightarrow c} f(x) = L$. Find the value of $\lim_{h \rightarrow 0} f(h+c)$.
5. Find $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$.
6. State first derivative theorem of local extreme values

(6x1=6)

PART B

II. Answer any seven questions. Each question carries 2 marks

7. Show that the value of $\int_0^1 \sin(x^2) dx$ cannot possibly be 2.
8. State Fundamental Theorem of Calculus (Part 1). Use fundamental theorem to evaluate

$$\frac{d}{dx} \left(\int_0^{\sqrt{x}} \cos t dt \right)$$

9. Using substitution method evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot \theta \operatorname{cosec}^2 \theta d\theta$

10. Find the length of the curve $y = \frac{4\sqrt{2}}{3} (x)^{\frac{3}{2}} - 1, 0 \leq x \leq 1$.

11. Define the surface area generated by revolving the curve about the x-axis.

12. If $f(x) = \frac{1}{x}, x_0 = 4, \epsilon = 0.05$ and $L = \frac{1}{4}$ find $\delta > 0$ such that

$$0 \leq |x - x_0| < \delta \Rightarrow |f(x) - L| < \epsilon.$$

13. Evaluate $\lim_{x \rightarrow 2^+} \frac{x-3}{x^2-4}$.

14. For the function $f(x) = \begin{cases} \frac{|x-4|}{x-4}, & x \neq 4 \\ 0, & x = 4 \end{cases}$ does $\lim_{x \rightarrow 4} f(x)$ exist

15. Show that the function $f(x) = x^3 + \frac{4}{x^2} + 7$ has exactly one zero in the interval $(-\infty, 0)$

16. Verify Mean Value theorem for $f(x) = x(x-1)(x-2)$ in the interval $[0, \frac{1}{2}]$.

(7x2=14)

PART C

III. Answer any five questions. Each question carries 6 marks

17. Given $\lim_{x \rightarrow c} f(x) = L$ and $\lim_{x \rightarrow c} g(x) = M$. Prove that $\lim_{x \rightarrow c} [f(x) + g(x)] = L + M$
18. Find c in the Mean Value Theorem $f(b) = f(a) + (b-a)f'(c)$ when $f(x) = x^3 - 3x^2 + 2x$, and $a=0, b=1/2$
19. Find the equation of tangent line and normal line at the point $(2, 4)$ to the curve $x^3 + y^3 - 9xy = 0$
20. Find the critical points of $f(x) = -x^3 + 12x + 5$, $-3 \leq x \leq 3$. Identify the intervals on which f is increasing and decreasing. Find the functions local and absolute extreme values.
21. Find the average value of $f(x) = \sqrt{4 - x^2}$ on $[-2, 2]$
22. Using limit of Riemann sums establish the equation $\int_a^b c \, dx = c(b-a)$.
23. The region between the curve $y = \sqrt{x}$, $0 \leq x \leq 4$ and the x -axis is revolved about the x -axis to generate a solid. Find its volume.
24. A pyramid 3m high has a square base that is 3m on a side. The cross-section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m on a side. Find the volume of the pyramid.

(5x6=30)

PART D

III. Answer any two questions. Each question carries 15 marks

25. State and prove Mean Value Theorem and hence verify Mean Value Theorem for the function $f(x) = \log x$ on the interval $[1, e]$
26. (a) If $y^3 - 3ax^2 + x^3 = 0$, then prove that $\frac{d^2y}{dx^2} + \frac{2a^2x^2}{y^5} = 0$
(b) Find the slope of the circle $x^2 + y^2 = 25$ at the point $(-4, 3)$
(c) Show that the $(2, 4)$ lies on the curve $x^3 + y^3 - 9xy = 0$. Then find the tangent and normal to the curve at the given point
27. Find the area between the graph of $y = -x^2 - 2x$ and x -axis over $[-3, 2]$
28. Find the surface area generated by revolving the curve $y = 2\sqrt{x}$, $1 \leq x \leq 2$ about the x -axis

(2x15=30)