TB165370E

Reg. No.: ....

Name : ....

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2018
(2016 Admission Regular & 2015 Admission Supplementary)
SEMESTER V - CORE COURSE (MATHEMATICS)
MT5B05B – REAL ANALYSIS I

Time: Three Hours **Maximum marks: 80**

PART A

I. Answer all questions. Each one carries 1 mark.

1. Consider the set $\{x: x \text{ is rational and } 0 \leq x < \pi\}$. Explain why this set necessarily has a supremum?
2. Find the infimum of the set $\left\{1 - \frac{1}{n^2} : n\epsilon\mathbb{N}\right\}$.
3. Give an example of a bounded set which is perfect.
4. What is the nature of convergence of the sesquence $\{n + (-1)^n n\}; n\epsilon\mathbb{N}$.
5. Define the distance between two nun-empty subsets A and B of a metric space (X, d).
6. Prove that every finite subset of a metric space is closed.

(6×1=6)

PART B

II. Answer any seven questions. Each one carries 2 marks.

7. Let m be the infimum of a set S and $'a'$ be a real number greater than m. Can $'a'$ be a lower bound of S. Why?
8. Give an examples of the sets:
   i) Finite bounded set.
   ii) Infinite bounded set.
9. Show that the set $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots\ldots\ldots\ldots\ldots, \frac{1}{n}\right\}$ is neither closed nor dense in itself.
10. If M and N are neighbourhoods of a point x, then show that $M \cap N$ is also a neighbourhood of x.
11. Prove that $\mathbb{N} \times \mathbb{N}$ is countable.
12. Define monotonic sequence. Give an example.
13. Find limit inferior and limit superior of the sequence $\{a_n\}$; where $a_n = \sin\frac{n\pi}{3}; n\epsilon\mathbb{N}$.
14. Give examples of sequences:
    i) A sequence havin limit whose range set doesn't have a limit point.
    ii) A sequence havin limit whose range set has a limit point.
15. Show that the subset $A = [0,1)$ of the metric space (X, d) where $X = [0,2)$ and d is the usual metric is an open set.
16. How many types of adherent points are there? Which are they?.

(7×2=14)

PART C

III. Answer any five questions. Each one carries 6 marks.

17. State and prove Archimedean property of real numbers.
18. State order completeness in $\mathbb{R}$. Prove that the set of natural numbers is order complete.

19. Prove that the derived set of a set is closed.
20. Prove that a countable union of countable sets is countable.
21. Show that:
    i) $\lim_{n\to\infty} a^{\frac{1}{n}} = 1$, if $a > 0$.
    ii) $\lim_{n\to\infty} n^{\frac{1}{n}} = 1$.
22. State and prove Sandwich theorem.
23. In any metric space(X, d), prove that:
    i) The intersection of an arbitrary family of closed sets is closed.
    ii) The union of a finite number of closed sets is closed.
24. Let (X, d) be any metric space and A be any non-empty subset of X, then show that $x\epsilon\bar{A}$ if and only if there exists a sequence $\{x_n\}$ in A such that $x_n \to x$ as $n \to \infty$..

(5×6=30)

PART D

IV. Answer any two questions. Each one carries 15 marks.

25. Prove the equivalence of Dedekind's form of completeness property and Order completeness property in $\mathbb{R}$.
26.
    i) State and prove Bolzanno Weirstrass theorem for sets.
    ii) Define a countable set. Prove that the set of real numbers is uncountable.
27.
    i) State and Prove Cauchy's first theorem on limits.
    ii) State and prove Cesaro's theorem.
28.
    i) Prove that every non-empty open set on the real line is the union of a countable collection of pointwise disjoint open intervals.
    ii) Let (X, d) be a metric space and $Y \subseteq X$, then show that a subset A of Y is open in (y, d_Y) if and only if there exists a set G open in (X, d) such that $A = G \cap Y$.

(2×15=30)