TB165380E

Reg. No.: ..............................

Name : ..................................

**B. Sc. DEGREE (C.B.C.S.S) EXAMINATION, OCTOBER 2018**
**(2016 Admission Regular & 2015 Admission Supplementary)**
**SEMESTER V- CORE COURSE (MATHEMATICS)**
**MT5B07B - ABSTRACT ALGEBRA**

**Time: Three Hours** **Maximum Marks: 80**

## PART A

**I. Answer all questions. Each question carries 1 mark.**

1. On Z define * by a * b=a-b. Check whether * is a binary operation on Z.
2. Compute the subgroup <3> of the group $Z_5$ under addition modulo 5.
3. Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 3 & 6 & 1 & 2 \end{pmatrix}$ on $\{1,2,3,4,5,6\}$ as a product of disjoint cycles
4. Define a normal subgroup of a group G.
5. Is the set of all complex numbers with usual addition and multiplication a ring,
6. Find the characteristic of the ring Z.

**(6x1=6)**

## PART B

**II. Answer any seven questions. Each question carries 2 marks.**

7. If G is a group with binary operation* and if a and b are any elements of G, then prove that the linear equations a * x =b has a unique solution in G
8. Write the proper subgroups of Klein four group..
9. Compute (1,4,5) (7,8) (2,1) (2,5,7) ..
10. Find all the cosets of 5Z of Z
11. If $\phi : G \rightarrow G'$ is a group homomorphism, then prove that Ker$\phi$ is a normal subgroup of G.
12. Prove that a group isomorphism maps identity onto identity and inverses onto inverses.
13. Find the number of elements in the cyclic sub group of $Z_{30}$ generated by 25.
14. Find $\phi(18)$ for $\phi : Z \rightarrow Z_{10}$ such that $\phi(1) = 6$
15. Prove that every field F is an Integral Domain.
16. Find all divisors of zero in $Z_6$.

**(7X2=14)**

## PART C

**III. Answer any five questions. Each question carries 6 marks.**

17. If H and K are subgroups of an abelian group G, then is H U K a subgroup of G?.
18. Prove that every permutation of a finite set A is a product of disjoint cycles.
19. Prove that a sub group of a cyclic group is cyclic.
20. Let G be a cyclic group with n elements and generated by a. Let $b \in G$ and $b = a^s$, then b generates a cyclic subgroup H containing $\frac{n}{d}$ elements, where d is the g.c.d of n and s.
21. Prove that any infinite cyclic group G is isomorphic to the group Z of integers under



(P.T.O)

addition.

22. Prove that a homomorphism $\phi$ of a group G is a one-one function iff Ker $\phi=\{e\}$.
23. Solve 3x=2 in the field $Z_7$.
24. a).Define a field.

    b).Prove that cancellation laws hold in a ring R if and only if R has no divisors of 0.

**(5x6= 30)**

## PART D

**IV. Answer any two questions. Each question carries 15 marks.**

25. a) Is the set of all n x n matrices a group under matrix addition and multiplication.

    b) Define a general linear group.

    c).Prove that intersection of two subgroups of a group G is a subgroup of G.
26. a) State and prove Cayley's Theorem.

    b) Prove that for $n \geq 2$, the number of even permutations in $S_n$ is same as the number of odd permutations.
27. a) State Fundamental Homomorphism theorem for groups.

    b) Let H be a normal subgroup of G. Then $\gamma : G \to G'$ given by $\gamma(x) = xH$ is a homomorphism with Kernel H.

    c) Find the order of (8, 4, 10) in the group $Z_{12} \times Z_{60} \times Z_{24}$.
28. a)Define a normal group.

    b) Let $\phi : G \to G'$ be a group homomorphism, let H= Ker$\phi$. Let $a \in G$. Then the set $\phi^{-1}[\{\phi(a)\}] = \{x \in G | \phi(x) = \phi(a)\}$ is the left coset aH of H.

    c) Let H be a normal subgroup of G. Then show that the cosets of H form a group G/H under the binary operation (aH)(bH)=(ab)H.

**(2x15=30)**