$\qquad$
Name : $\qquad$

## B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, JANUARY 2019

(2016 Admission Supplementary)
SEMESTER V- CORE COURSE (MATHEMATICS)
MT5B06B - DIFFERENTIAL EQUATIONS
Time: Three Hours
Maximum Marks: 80

## PART A

I. Answer all. Each one carries 1 mark.

1. Define Bernoulli differential equation.
2. Find the integrating factor of the equation $\frac{d x}{d y}+\frac{2}{y} x=y^{2}$.
3. Write the UC set of the function $x^{2} e^{4 x} \sin x$.
4. Solve $\frac{d^{2} y}{d x^{2}}-3 \frac{d y}{d x}+2 y=0$.
5. Define ordinary point of a second order differential equation.
6. The direction cosines of the normal to the surface $z=f(x, y)$ at the point $(x, y, z)$ are

## PART B

II. Answer any seven questions. Each one carries $\mathbf{2}$ marks.
7. Solve $(3 x+2 y) d x+(2 x+y) d y=0$.
8. Solve $\frac{d y}{d x}+\frac{y}{x}=x^{2}$.
9. Write the UC set of the functions $x^{3} e^{5 x} \sin x, x^{5} e^{2 x}$.
10. Show that $e^{-x}, e^{3 x}, e^{4 x}$ are linearly independent solution of the differential equation
$\frac{d^{3} y}{d x^{3}}-6 \frac{d^{2} y}{d x^{2}}+5 \frac{d y}{d x}+12 y=0$.
11. Solve $\frac{d^{2} y}{d x^{2}}-4 \frac{d y}{d x}+13 y=0$.
12. Locate and classify the singular points of $\left(x^{3}+x^{2}\right) \frac{d^{2} y}{d x^{2}}+\left(x^{2}-2 x\right) \frac{d y}{d x}+4 y=0$.
13. Show that $J_{-p}(x)=(-1)^{p} J_{p}(x)$, if $p$ is an integer.
14. Evaluate:
i) $(D-1)(D-2)\left(t^{2}+e^{t}\right)$.
ii) $\left(D^{2}+3 D\right)\left(e^{3 t}+\cos t\right)$.
15. Form the partial differential equation $F\left(x^{2} y^{2} z^{2}, x y z\right)=0$.
16. Find the equation of the tangent plane to the surface $x^{2}+y^{2}+z^{2}=1$ at the point $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}\right)$.

## PART C

III. Answer any five questions. Each one carries 6 marks.
17. Solve $(x+2 y+3) d x+(2 x+4 y-1) d y=0$.
18. Solve $\frac{d y}{d x}+y=x y^{3}$.
19. Solve $\frac{d^{2} y}{d x^{2}}+4 y=\sec ^{2} 2 x$.
20. Given that $y=x$ is a solution of $\left(x^{3}-x+1\right) \frac{d^{2} y}{d x^{2}}+\left(x^{2}+x\right) \frac{d y}{d x}+(x+1) y=0$. Find a linearly independent solution by reducing the order. Find the general solution.
21. Solve $x^{2} \frac{d^{2} y}{d x^{2}}-4 x \frac{d y}{d x}+4 y=4 x^{2}-6 x^{3}$.
22. Find the power series solution of the differential equation $\frac{d^{2} y}{d x^{2}}+x \frac{d y}{d x}+y=0$.
23. $J_{p}(x)$ is the solution of the Bessel's equation of order $p$. Then show that :
i) $\frac{d}{d x}\left(x^{p} J_{p}(x)\right)=x^{p} J_{p-1}(x)$
ii) $\frac{d}{d x}\left(x^{-p} J_{p}(x)\right)=x^{-p} J_{p+1}(x)$
24. Find the general integral of $y^{2} p-x y q=x(z-2 y)$.
( $5 \times 6=30$ )

## PART D

## IV. Answer any two questions. Each one carries 15 marks.

25. Solve the differential equation $(x-2 y-3) d x+(2 x+y-1) d y=0$.
26. Solve $\frac{d^{2} y}{d x^{2}}-3 \frac{d y}{d x}+2 y=x^{2} e^{x}$.
27. Find the general solution of $\left(x^{2}-1\right) y^{\prime \prime}-2 x y^{\prime}+2 y=\left(x^{2}-1\right)^{2}$.
28. Find the general solution of the linear system of equations:

$$
\begin{aligned}
& 2 \frac{d x}{d t}-2 \frac{d y}{d t}-3 x=t \\
& 2 \frac{d x}{d t}+2 \frac{d y}{d t}+3 x+8 y=2
\end{aligned}
$$

( $2 \times 15=30$ )

