TB246970X

Reg. No :..............................

Name :..................................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, MARCH 2024

2021 ADMISSIONS REGULAR

SEMESTER VI - CORE COURSE (MATHEMATICS)

MT6B10B18 - Complex Analysis

Time : 3 Hours

Maximum Marks : 80

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. When do we say that a given set is bounded.
2. Define a connected set.
3. Show that $z_n = (-1)^n(1+i)\frac{n-1}{n}$ has two accumulation points.
4. Find the derivative of $f(z)=\frac{z-1}{2z+1}(z\neq -1/2)$.
5. Evaluate $\lim\limits_{z\to i}\frac{iz^3-1}{z+i}$.
6. Define a simply connected domain. Is the region between two concentric circles simply connected or multiply connected?
7. State Cauchy Goursat theorem.
8. Write the equation of a line segment in the form u+iv joining 0 and 1+i.
9. Find the series representation of $\frac{1}{1-z}.(|z|<1)$
10. State Laurent's theorem.
11. Evaluate the residue of $e^{\frac{1}{z^2}}$ at z=0.
12. Define simple pole. Give example.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Express $f(z)=i(z^3+C)$ in the form u+iv and show that u and v are harmonic functions.
14. If a function is continuous and non zero at a point z_0 , then show that $f(z)\neq 0$ through out some neighbourhood of that point.
15. Suppose that $f(z) = u + iv$ and its conjugate $\overline{f(z)}=u-iv$ are analytic in a domain D then show that f(z) is constant in D.
16. Evaluate $\int_C f(z)dz$ where $f(z) = \frac{z^2+1}{z^2-1}$ where . $c\colon |z| = 2.$

17. Evaluate a. $\int_{OA} f(z)dz+\int_{AB} f(z)\,dz$, where $f(z)=y-x-3ix^2$ and OA is the line segment from 0 to i and AB is the line segment from i to 1+i.

    b. $\int_{OB} f(z)dz$ where OB is the line segment joining 0 to 1+i and f(z) is the same as above.

18. Evaluate $\int_C \frac{z^2-1/3}{z^3-z}dz$ where C is $|z-1/2|=1$

19. Show that when $z \neq 0$ $z^3\cosh\left(\frac{1}{z}\right)=\frac{z}{2}+z^3+\sum_{n=1}^{\infty}\frac{1}{(2n+2)!}\frac{1}{z^{(2n-1)}}$, $0<|z|<\infty$

20. Find the order of the pole and its residue at z=2 of $\frac{z^2-2z+3}{z-2}$.

21. Evaluate $\int_C \frac{dz}{z(z-2)^4}$, where C is the positively oriented circle $|z-2|=1$.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. a. Show that a set is open if and only if each point in S is an interior point.
    b. Prove that if a set contains each of its accumulation point, then it must be a closed set.
    c. Is the set consisting of all points z such that $1<|z|<2$ connected?

23. State and prove maximum modulus principle.

24. Find the series representation of $f(z)=\frac{-1}{z^2-3z+2}$ in the regions a. $|z|<1$ b. $1<|z|<2$ c. $|z|>2$

25. Use residue to evaluate $\int_0^{\infty}\frac{x^2dx}{x^6+1}$.