18.4

TB246980E

Reg. No :..............................

Name :..................................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, MARCH 2024
2021 ADMISSIONS REGULAR
SEMESTER VI - CORE COURSE (MATHEMATICS)
MT6B12B18 - Linear Algebra

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Give an example of a two dimensional vector space.
2. Determine whether $\{t, 2\}$ of $\mathbb{P}^1$ is linearly independent ?
3. Define the rank of a linear transformation.
4. A 2x2 matrix A is known to have the eigen values -3 and 4. Calculate the eigen values of A^2.
5. Define the image of a matrix.
6. Define a diagonalizable matrix.
7. Determine whether $A = \begin{bmatrix} 2 & 0 & 0 \\ -3 & 3 & 0 \\ 2 & -1 & 4 \end{bmatrix}$ is diagonalizable.
8. The determinant of a 4x4 matrix is 144 and two of its eigen values are known to be -3 and 2. What can be said about the remaining eigen values ?
9. Establish that a matrix is singular if and only if it has a zero eigen value.
10. State Cauchy-Schwarz inequality.
11. Normalize the vector $\bar{v} = \begin{bmatrix} 20 & -5 \end{bmatrix}^T$
12. Determine whether the set of vectors $\{\bar{u}, \bar{v}, \bar{w}\}$ in $\mathbb{R}^3$ is an orthogonal set of vectors where $\bar{u} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \bar{v} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \\ 0 \end{bmatrix}, \bar{w} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Prove that if a matrix B is obtained from a matrix A by an elementary row operation, then the row space of A is same as the row space of B.
14. For any vector $\bar{w}$ in a vector space v, show that $-1 \odot \bar{w} = -\bar{w}$
15. Calculate the coordinate representation of $\begin{bmatrix} 1 & 3 \end{bmatrix}$ with respect to the basis $\{\begin{bmatrix} 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \end{bmatrix}\}$ of $\mathbb{R}^2$
16. Prove that the transition matrix from C to D, where both C and D are bases for the same finite dimensional vector space is invertible and its inverse is the transition matrix from D to C.
17. Find the transition matrices between two bases $G = \{t + 1, t - 1\}$ and $H = \{2t + 1, 3t + 1\}$ for $\mathbb{P}^1$
18. Find the matrix representation of the linear transformation $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ defined by $T\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 11a + 3b \\ -5a - 5b \end{bmatrix}$ with respect to the basis $E = \{\begin{bmatrix} 3 & -1 \end{bmatrix}^T, \begin{bmatrix} 1 & -5 \end{bmatrix}^T\}$.

19. Prove that similar matrices have the same eigen values.

20. Establish that the product of all the eigen values of a matrix equals the determinant of the matrix.

21. Establish that if $\bar{x}$ and $\bar{y}$ are orthogonal vectors in $\mathbb{R}^n$, then $||\bar{x}+\bar{y}||^2 = ||\bar{x}||^2 + ||\bar{y}||^2$

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. Describe the span of the vectors in the set

$$\mathbb{R} = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$$

23. State and prove rank-nullity theorem.

24. Let T be a linear transformation from an n-dimensional vector space V into W and let $\{\bar{v}_1, \bar{v}_2, \ldots \bar{v}_k\}$ be a basis for the kernel of T. If this basis is extended to a basis $\{\bar{v}_1, \bar{v}_2, \ldots \bar{v}_k, \bar{v}_{k+1}, \ldots \bar{v}_n\}$ for V, then prove that $\{T(\bar{v}_{k+1}), T(\bar{v}_{k+2}), \ldots T(\bar{v}_n)\}$ is a basis for the image of T.

25. Determine whether the linear transformation $T : P^1 \mapsto P^1$ defined by $T(at+b) = (a+2b)t + (4a+3b)$ can be represented by a diagonal matrix.