TB246709P

Reg. No :..............................

Name :..............................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, MARCH 2024

2021 ADMISSIONS REGULAR

SEMESTER VI - CORE COURSE (MATHEMATICS)

MT6B09B18 - Real Analysis –II

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. State Cauchy's general principle of convergence for infinite series.
2. Give an example of an alternating series that is convergent.
3. Illustrate with the help of a counter-example that Cauchy's root test fails in analyzing the nature of convergence of some series.
4. State Fixed point theorem
5. Is the function f(x) = x - |x| continuous ? Justify.
6. Check whether the function f defined on **Z** , the set of integers as $f(x) = \frac{8}{9x-1}$ a continuous function ? Justify your answer.
7. Define upper Darboux sum of a bounded function f over [a,b]
8. State Darboux's condition of integrability of a bounded function.
9. Compute the lower Darboux sum L(P, f) of the function f(x) = 2x + 3 for the partition P = {2, 2.2, 2.9, 3} of [2, 3].
10. If P^* is a refinement of partition P, the show that $U(P^*, f) \leq U(P, f)$.
11. State Dirichlet's test for uniform convergence
12. Check the uniform convergence of the series whose n^{th} term is $(0.5)^n \cos n^2 x$.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. State and prove Raabe's test.
14. Check the convergence of the series whose n^{th} term is given by $(n^3+1)^{1/3} - n$
15. If a function f(x) is continuous on [a,b] and f(a) ≠ f(b), then prove that it assumes every value between f(a) and f(b).
16. Discuss the continuity of the function f defined on R by

$$f(x) = \begin{cases} -x^2, & \text{if } x \leq 0 \\ 5x-4, & \text{if } 0 < x \leq 1 \\ 4x^2 - 3x & \text{if } 1 < x < 2 \\ 3x+4 & \text{if } x \geq 2 \end{cases}$$

at the points x = 0, 1 and 2 . .

17. If a function f is monotonic on [a,b], then show that it is integrable on [a,b].
18. Compute the upper integral and lower integral of the function $f(x) = \begin{cases} 0, & \text{when x is rational} \\ 1, & \text{when x is irrational} \end{cases}$ and hence prove that f is not integrable on any interval.

19. If a refinement P^* of the partition P of [a, b] contains p points more than P, and $f(x) \le k \ \forall x \in [a, b]$, then prove that $L(P, f) \le L(P^*, f) \le L(P, f) + 2pk\mu$ where μ is the norm of P.

20. State and prove Cauchy's criterion for uniform convergence of a sequence of functions .

21. Test for uniform convergence of $\{ f_n \}$ where $f_n(x) = \dfrac{x}{1+nx^2}$ *for all real x.*

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. State and prove Leibnitz test for checking the convergence of an alternating series

23. (a) If a function f is continuous on [a,b] and f(a).f(b) < 0, then prove that there exist at least one point c in (a, b) such that f(c) = 0.

(b) Discuss the kind of discontinuity if any of the function $f(x) = \dfrac{x-[x]}{x}$ at x = 2

24. State and prove a necessary and sufficient condition for the integrability of a bounded function on a closed interval.

25. (a) If $\sum_{n=1}^{\infty} a_n x^n$ is a series which converges for all values of x where $|x| < R$, then prove that $\sum a_n R^n$ is uniformly convergent in [0, R] iff $\sum a_n R^n$ is convergent.

(b)State and prove Weierstrass M- test for uniform convergence of a series of functions.