TB165385F

Reg. No.: ....

Name : ....

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, JANUARY 2019

(2016 Admission Supplementary)

SEMESTER V- CORE COURSE (MATHEMATICS)

MT5B08B - GRAPH THEORY

Time: Three Hours **Maximum Marks: 80**

PART A

I Answer all the questions. Each question carries 1 mark.

1. Let G be a graph in which there is no pair of adjacent edges. What can you say about the degrees of the vertices in G?
2. What is the smallest integer n such that the complete graph K_n has at least 500 edges?
3. Draw the Petersen graph.
4. State Cayley's Theorem.
5. Define Hamiltonian graph.
6. Define a directed graph.

(6 x 1 = 6)

PART B

II Answer any seven questions. Each question carries two marks.

7. Prove that in a k-regular graph G where k is a odd number, the number of edges is a multiple of k.
8. Prove that it is impossible to have a group of thirteen people in a conference where each person knows exactly 5 of others.
9. Prove that if G is a self-complementary graph with n vertices, then n is either $4t$ or $4t+1$ for some integer t.
10. Prove that any tree with at least 2 vertices is a bipartite graph.
11. Let G be an acyclic graph with n vertices and k connected components then show that G has n-k edges.
12. State Hall's Marriage Theorem.
13. Define matching and perfect matching.
14. Draw all possible tournaments on 4 vertices.
15. State Camion Theorem.
16. Draw the de Burjin Diagram $D_{2,3}$.

(7 x 2 = 14)

PART C

III Answer any six questions. Each question carries five marks.

17. Draw the corresponding graph for the following adjacency matrix

$$A[G] = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

18. Prove that an edge e of a graph G is a bridge if and only if it is not part of any cycle in G.
19. Let G be a graph with n vertices. Then show that the following three statements are equivalent
a) G is a tree,
b) G is an acyclic graph with n-1 edges,
c) G is a connected graph with n-1 edges.
20. Let G be a k-regular bipartite graph with $k > 0$. Then show that G has a perfect matching.
21. Prove that a matching M in a graph G is a maximum matching if and only if G contains no M-augmenting path.
22. Let v be any vertex having maximum out degree in the tournament T. Then prove that for every vertex w of T there is a directed path from v to w of length at most 2
23. State and prove Redei's Theorem.
24. Prove that an Euler digraph is strongly connected.

(6 x 5 = 30)

PART D

IV Answer any two questions. Each question carries 15 marks.

25. Let G be a nonempty graph with at least two vertices. Then prove that G is bipartite if and only if it has no odd cycles.
26. State and prove Whitney's Theorem.
27. State and prove Dirac's Theorem.
28. Let D be a weakly connected digraph with at least one arc. Then show that D is Euler if and only if $od(v) = id(v)$ for every vertex v of D.

(2 x 15 = 30)