TB245775O

Reg. No :..............................

Name :..................................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, FEBRUARY 2024

2021 ADMISSIONS SUPPLEMENTARY (SAY)

SEMESTER V - CORE COURSE (MATHEMATICS)

MT5B06B18 - Real Analysis-I

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Prove or disprove : A bounded set always has the greatest element and the smallest element.
2. Determine the greatest element and the infimum of the set $\{x \in Q: 2.5 < x \leq 3.8\}$.
3. Boundedness is not necessary in order for an infinite set S to have a limit point. Justify with an example.
4. Prove that superset of a neighborhood of a point x is also a neighborhood of x.
5. Obtain the derived set of the following sets :
   (a) (8, 9)
   $(b) \left\{1, -1, 1\frac{1}{2}, -1\frac{1}{2}, 1\frac{1}{3}, -1\frac{1}{3}, \ldots\right\}$
6. If S and T are subsets of R, then give an example to show that $(S \cap T)'$ and $S' \cap T'$ may not be equal where S' is the derived set of S and T' is the derived set of T.
7. Define a monotonic sequence. Give an example of a sequence which is not monotonic.
8. Prove that a sequence cannot converge to more than one limit.
9. Check the nature of convergence of the sequence $\left\{1 - \frac{1}{n}\right\}$
10. Show that for any real number x, $\lim \frac{x^n}{n!} = 0$.
11. Define a bounded metric space. Give an example.
12. Check whether the function $d : R \times R \to R$ defined by d(x, y) = max {xy, 0} is a metric on R where R is the set of real numbers.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Let K be the infimum of a set S and 'a' be a real number greater than K. Can 'a' be a lower bound of S. Why?
14. Give an example of the following : a) A set with both the supremum and the infimum. b) A set with the supremum and without the infimum. c) A set with the supremum and without the greatest element. d) A set with the infimum and without the smallest element. e)A set without the smallest element and the greatest element.
15. Prove or disprove : Union of an arbitrary collection of closed sets is a closed set.
16. Prove or disprove : The union of an arbitrary family of open sets is an open set.
17. State and prove Bolzano-Weirstrass theorem for sequences.
18. Show that the sequence $\{(a_n)^{1/n}\}$ converges and find its limit where $a_n = \frac{(3n)!}{(n!)^2}$
19. Show that $\{S_n\}$ where $S_n = (1 + 1/n)^n$ is convergent and its limit lies between 2 and 3 .
20. In any metric space (X, d), show that the intersection of a finite number of open sets is open.