TB245820J

Reg. No :...............................

Name :..................................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, FEBRUARY 2024

2021 ADMISSIONS SUPPLEMENTARY (SAY)

SEMESTER V - CORE COURSE (MATHEMATICS)

MT5B07B18 - Differential Equations

Time : 3 Hours

Maximum Marks : 80

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. **Determine whether the equation** $(x^{-2}+y^{-2})dx+(2y^2x+1)y^{-3}dy=0$ **is exact.**
2. **Determine the integrating factor of** $(2x+tany)\,dx+\left(x-x^2tany\right)dy=0.$
3. **Solve** $tan\theta\,dr+2rd\theta=0.$
4. Evaluate the general solution $4\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+y=0.$
5. The general solution of a differential equation is given by $e^{3x}\,(Asinx+Bcosx)$. Find A and B if $y(0)=-3, y'(0)=-1.$
6. Determine whether $sin\theta$ and $cos\theta$ are linearly independent.
7. Evaluate the wronskian and check whether e^x and e^{2x} are linearly independent.
8. Evaluate the value of $\Gamma\left(\dfrac{1}{2}\right)$.
9. Express the standard form of the Frobenius series solution of a differential equation about a regular singular point.
10. Evaluate the value of $\Gamma\,(5)$.
11. Write the partial differential equation of $z=y^2+2f\left(\dfrac{1}{x}+lny\right)$.
12. Write the partial differential equation of $z=xy+f(x^2+y^2)$.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Determine the oblique trajectories that intersect the family of straight lines y = cx at an angle 45^0.
14. *Solve* $(x+2y+3)dx+(2x+4y-1)dy=0.$
15. Solve $x^3\dfrac{d^3y}{dx^3}-4x^2\dfrac{d^2y}{dx^2}+8x\dfrac{dy}{dx}-8y=4lnx$
16. Solve $\dfrac{d^2y}{dx^2}-3\dfrac{dy}{dx}+2y=4x^2.$
17. Solve the indicial equation of $2x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}+2y=0.$
18. Solve $\dfrac{dx}{dt}+\dfrac{dy}{dt}+2y=sint$,

$$\frac{dx}{dt} + \frac{dy}{dt} - x - y = 0.$$

19. Solve the indicial equation of $2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0.$

20. Find the solution of the partial differential equation $(y(x+y)+az)p + (x(x+y)-az)q = z(x+y)$.

21. Solve the partial differential equation $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. **a) Establish that if M(x,y)dx +N(x,y)dy=0 is a homogeneous differential equation, then the change of the variable y=vx reduces the above equation to a variable separable differential equation in v and x.**

b) Solve $\frac{dy}{dx} = \frac{4x^3y^2 - 3x^2y}{x^3 - 2x^4y}$.

23. **a) Solve the differential equation** $\frac{d^2y}{dx^2} + y = \cot x$ **by the method of variation of parameters.**

b) If $y = x$ **and** $y = x^2 - 1$ **are solutions of the corresponding homogeneous equation of** $(x^2+1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 6(x^2+1)^2$ **, then compute the general solution.**

24. Use Frobenius method to solve $x^2\frac{d^2y}{dx^2} + (x^2 - 3x)\frac{dy}{dx} + 3y = 0.$

25. **a) Find the integral curves of** $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$

b) If $u = f(x+iy) + g(x-iy)$ **then show that u satisfies Laplace's equation.**