TB165370F

Reg. No.: ....

Name : ....

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, JANUARY 2019
(2016 Admission Supplementary)
SEMESTER V - CORE COURSE (MATHEMATICS)
MAT5B05TB – REAL ANALYSIS I

Time: Three Hours **Maximum marks: 80**

PART A

I. Answer all questions. Each one carries 1 mark.

1. State order completeness in $\mathbb{R}$ in terms of infimum.
2. The set $\left\{\frac{2}{n} : n\epsilon\mathbb{N}\right\}$ is unbounded. True/False. Justify.
3. Give an example of a bounded set with exactly two limit points.
4. Give the nature of convergence of the sequence $\{n + (-1)^n\}; n\epsilon\mathbb{N}$.
5. Give an example of a metric space.
6. Define a dense subset of a metric space.

(6×1=6)

PART B

II. Answer any seven questions. Each one carries 2 marks.

7. Find the supremum and infimum of the set $\{x\epsilon\mathbb{Q}: 1 < x < 2\}$.
8. Prove that $\mathbb{N}$ is order complete.
9. Give examples of two sets L and U satisfying Dedekind's form of completeness property.
10. Show that every open interval is an open set.
11. Boundedness is not necessary in order for an infinite set S to have a limit point. Justify with an example.
12. Prove that the set $\{1, 4, 9, 16, \dots\dots\dots\dots\}$ is countable.
13. Prove that a sequence cannot converge to more than one limit.
14. Find the limit of the sequence $\{x_n\}$ where $x_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \dots\dots\dots\dots + \frac{1}{(2n)^2}$ using Cauchy's first theorem on limits.
15. If $A \subseteq B$, then prove that $\bar{A} \subseteq \bar{B}$.
16. Give an example of a countable family of closed subsets of $\mathbb{R}$ whose union is not closed.

(7×2=14)

PART C

III. Answer any five questions. Each one carries 6 marks.

17. Prove that every infinite bounded set has a limit point.
18. Prove that a countable union of countable sets is countable. Also prove the countability of

rational numbers using this result.

19. Define interval. Check whether the following sets are intervals and give reasons:

i) $\{1,3,5,7\}$

ii) $\{x \in \mathbb{Q} : 1 < x < 3\}$

iii) $\{x \in \mathbb{R} : -1 < x < 0\}$

iv) $\{x \in I : 0 < x < 5\}$

20. If $S \subseteq T \subseteq \mathbb{R}$, where $S \neq \phi$, then show that:

i) If T is bounded abov, then $\sup S \leq \sup T$.

ii) If T is bounded below, then $\inf T \leq \inf S$.

21. Show that:

i) $\lim_{n\to\infty} a^{\frac{1}{n}} = 1$, if $a > 0$.

ii) $\lim_{n\to\infty} n^{\frac{1}{n}} = 1$.

22. Prove that the sequence $\{s_n\}$ defined by the recursion formula $s_{n+1} = \sqrt{7 + s_n}$; $s_1 = \sqrt{7}$ converges to the positive root of $x^2 - x - 7 = 0$.

23. In any metric space(X, d), prove that:

i) The union of an arbitrary family of open sets is open.

ii) The intersection of a finite number of open sets is open.

24. Prove that the cantor set is a perfect set.

(5×6=30)

PART D

IV. Answer any two questions. Each one carries 15 marks.

25. State and prove Cantor's intersection theorem using the concept of metric.

26.

i) State and prove Cauchy's general principle of convergence.

ii) Define Cauchy sequence. Show that the sequence $\{s_n\}$ where $s_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots\dots\dots + \frac{1}{n}$ cannot converge.

27.

i) State and prove Archimedian property of real numbers.

ii) Prove that every open interval (a, b) contains a rational number.

28. Prove that:

i) The derived set of a bounded set is bounded.

ii) The derived set S' of a bounded infinite set $S \subseteq \mathbb{R}$ has the smallest and the greatest members.

(2×15=30)