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Reg. No :..............................

Name :..................................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, MARCH 2024

2022 ADMISSIONS REGULAR

SEMESTER IV - COMPLEMENTARY COURSE 1 (MATHEMATICS FOR PHYSICS & CHEMISTRY)

MT4C01B18 - Fourier Series, Partial Differential Equations, Numerical Analysis and Abstract Algebra

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Write the equation to find half range Fourier Cosine series.
2. Express Cosine Series and Sine series in terms of Power series.
3. Find out the Bessel functions $J_0(x)$ and $J_1(x)$.
4. Write the equation to find the tangent plane at a point P (x,y,z) to the surface S whose equation is F(x,y,z)=0.
5. Form a partial differential equation by eliminating the constants a and b from the equation $2z = (ax + y)^2 + b$.
6. Find an integral curve of the equations : $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{(x+y)z}$
7. Write Newton Raphson formula for finding an approximate root of f(x)=0
8. Is $5x^2 - 7x + 9x^6 + sin(x) = 34$ a transcendental equation? Justify your answer.
9. Define Symmetric group on n letters
10. If a and b are any two elements of a group G then show that $(a * b)^{-1} = b^{-1} * a^{-1}$
11. Is the group (Z_6 , $+_6$) cyclic? Justify your answer.
12. Find the order of 5 in Z_6

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Find the Fourier series of f given by $f(x) = x, -\pi < x < \pi$ and $f(x) = f(x + 2\pi)$ $\forall x \in R$.
14. Derive: $J_0'(x) = -J_1(x)$
15. Solve : $y' = 2xy$.
16. Obtain Fourier Sine series for the function $f(x) = c,\ x \in [0, \pi]$. Also deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$
17. Show that the family of spheres $x^2 + y^2 + (z - c)^2 = a^2$ satisfies the first order PDE $yp - xq = 0$.
18. Find a root of the equation 2x =cos x +3 correct to three decimal places by iteration method
19. Consider the map $\phi : Z \to R$ under addition defined by $\phi(x) = 4x$. Check whether ϕ is a group homomorphism.
20. Show that $< \mathbb{R}, +, . >$ is a ring where '+' is the usual addition, ' . ' is the usual multiplication and $\mathbb{R}$ is the set of real numbers.

21. Check whether Z, the set of integers is an abelian group with respect to addition?

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. Find the two half range expansions of $f(x) = x;\ 0 < x < L$.

23. (a) Form the partial differential equation by eliminating the constants from the equation $z = (x + a^2)(y + b^2)$.

(b) Form the partial differential equation by eliminating the arbitraty function from $f(x + y + z, x^2 + y^2 + z^2) = 0$.

(c) Find the integral curves of the equation $\dfrac{dx}{y + zx} = \dfrac{dy}{-(x + yz)} = \dfrac{dz}{x^2 - y^2}$.

24. Find the real root of $x^3 - 7x^2 + 10x - 2 = 0$ using Quotient Difference method

25.

(a). Define $*$ on Q^+ by $a * b = \dfrac{ab}{2}$. Show that Q^+ under the operation $*$ is an abelian group.

(b). Show that the set G ={1,-1, i, -i} forms an abelian group with respect to multiplication.