16.4

TB244356E

Reg. No :..............................

Name :..................................

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, MARCH 2024

2022 ADMISSIONS REGULAR

SEMESTER IV - CORE COURSE (MATHEMATICS)

MT4B04B18 - Vector Calculus, Theory of Equations and Matrices

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Define a binormal vector.
2. Find the parametric equation for a line through (1,1,1) parallel to z- axis.
3. Find an equation for the plane through P(2,4,5) and perpendicular to the line $x=5+t,\ y=1+3t,\ z=4t.$
4. Define potential function.
5. Determine whether the differential form $yzdx + xzdy + xydz$ is exact.
6. Determine whether $\bar{\mathbf{F}} = (2x-3)\bar{i} - z\bar{j} + (\cos z)\bar{k}$ is conservative.
7. Form an equation whose roots are 2 and 3+i.
8. Form an equation whose roots are the negatives of the roots of the equation $x^4-4x^3+6x^2-x+2=0$
9. Find the equation whose roots are two less than the roots of the equation $x^4-5x^3+7x^2-4x+5=0$
10. Define the rank of a matrix.
11. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$
12. Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{pmatrix}$

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Find the distance from (2,-3,4) to the plane $x+2y+2z=13.$
14. Write the acceleration in the form $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$, $r(t) = (\cos t + t\sin t)\vec{i} + (\sin t - t\cos t)\vec{j}, t>0.$
15. Integrate $f(x,y) = \dfrac{x^3}{y}$ over the curve $C : y = \dfrac{x^2}{2}, 0 \le x \le 2$
16. Find the surface area of the cone $z = \sqrt{x^2+y^2}$
17. Integrate $g(x,y,z) = xyz$ over the surface of the cube cut from the first octant by the planes $x=1, y=1$ and $z=1$
18. Solve the equation $x^4-2x^3+4x^2+6x-21=0$ given that two of its roots are equal in magnitude and opposite in sign
19. Solve $x^4-2x^3-3x^2+4x-1=0$ given that the product of two of its roots is unity.
20. Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$

21. Verify Cayley - Hamilton Theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. Find T, N, B, κ, τ for the space curve $r(t) = 3\sin t\,\vec{i} + 3\cos t\,\vec{j} + 4t\vec{k}$.

23. Find the circulation of the field $\vec{F} = (x^2 - y)\vec{i} + 4z\vec{j} + x^2\vec{k}$ around the curve C in which the plane z=2 meets the cone $z = \sqrt{x^2 + y^2}$.

24. Show that on diminishing the roots of the equation $6x^4-43x^3+76x^2+25x-100=0$ by 2 it becomes a reciprocal equation and hence solve it.

25. a) Using Cayley Hamilton Theorem, show that $A^3 - 6A^2 + 11A - 6I = 0$, where $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$, and hence find A^{-1}.

b) Find the eigen values and the corresponding eigenvectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.