TB165380F

Reg. No.: ...............................

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## B. Sc. DEGREE (C.B.C.S.S) EXAMINATION, JANUARY 2019
## (2016 Admission Supplementary)
## SEMESTER V- CORE COURSE (MATHEMATICS)
## MT5B07B - ABSTRACT ALGEBRA

**Time: Three Hours** **Maximum Marks: 80**

## PART A

**I. Answer all questions. Each question carries 1 mark.**

1. Give an example of an Abelian group of order four which is not cyclic.
2. Find the orderof $\langle\sigma\rangle$, the cyclic group generated by $\sigma$ in $S_5$ where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 2 & 5 & 1 \end{pmatrix}$.
3. Find the Kernel of $\phi : Z \rightarrow Z_5$ defined by $\varphi(m) = r$, where r is the remainder when m is divided by 5.
4. Find all cosets of the subgroup 4Z of Z.
5. Define ring homomorphism.
6. Find all the units in the ring $Z_{10}$..

**(6x1=6)**

## PART B

**II. Answer any seven questions. Each question carries 2 marks.**

7. Find the generators of the group $Z_6$ under $+_6$.
8. Check whether the set G={1,- 1, i, -i} forms an abelian group with respect to multiplication.
9. Find the number of generators of cyclic group of order 8.
10. Find the orbits of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 2 & 6 & 1 & 3 \end{pmatrix}$.
11. Find the number of odd permutations in the group $S_5$.
12. Prove that an isomorphism maps the identity onto the identity and inverses onto inverses
13. State the fundamental homomorphism theorem.
14. Prove that, a group homomorphism $\phi$ is one-one if and only if Kernel of $\phi$ is {e}.
15. Define an Integral Domain.
16. Solve $x^2 - 5x + 6 = 0$ in $Z_{12}$.

**(7x2=14)**

## PART C

**III. Answer any five questions. Each question carries 6 marks.**

17. Show that every group G with identity e and such that $x * x = e$ for all $x \in G$ is abelian.
18. Prove that identity element and inverse of each element are unique in a group G.
19. Prove that every group of prime order is cyclic.
20. Show that the number of even permutations in $S_n$ is the same as the number of odd

permutations.

21. Let $\phi : G \rightarrow G'$ be a group homomorphism, let H= Ker$\phi$. Let $a \in G$. Then the set $\phi^{-1}[\{\phi(a)\}] = \{x \in G | \phi(x) = \phi(a)\}$ is the left coset aH of H.
22. Let H be a normal subgroup of G. Then show that the cosets of H form a group G/H under the binary operation (aH)(bH)=(ab)H.
23. Prove that in the ring $Z_n$, the divisors of 0 are precisely those numbers that are not relatively prime to n.
24. Prove, if p is a prime, then $Z_p$ is a field.

**(5x6= 30)**

**PART D**

**IV. Answer any two questions. Each question carries 15 marks.**

25. a) Let G be a cyclic group with generator a. If the order of G is infinite, then G is isomorphic to (Z,+). If G has finite order n, then G is isomorphic to ($Z_n$, $+_n$).
    b) Find the cyclic subgroup generated by $< i >$ of the group of non-zero complex numbers under multiplication.
26. a) Let A be any nonempty set and let $S_A$ be the collection of all permutations of A. Prove that $S_A$ is a group under permutation multiplication.
    b). State and prove Cayley's Theorem.
27. a) Let H be a subgroup of G . Let the relation $\sim$ be defined on G by a ~ b if and only if $a^{-1}b \in H$ . Show that ~ is an equivalence relation on G.
    b) State and prove Lagrange's Theorem.
28. a). Prove that a factor group of a cyclic group is cyclic
    b). Prove that M is a maximal normal subgroup of G iff $G/M$ is simple.

**(2x15=30)**