

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, NOVEMBER 2024

2018, 2019, 2020, 2021, 2022 ADMISSIONS SUPPLEMENTARY

SEMESTER III - CORE COURSE (CQA) (STATISTICS)

ST3B03B18 Probability Distributions

Time : 3 Hours

Maximum Marks : 80

Part A

I. Answer any Ten questions. Each question carries 2 marks

(10x2=20)

1. A random variable X has p.d.f. $f(x) = 2^{-x}$; $x = 1, 2, 3, \dots$, find mode of the distribution.
2. Write any two limitations of mgf.
3. For a discrete random variable X , show that $E(aX + b) = aE(X) + b$.
4. Write any two properties of expectation of a random variable.
5. If $X \sim P(\lambda)$, find $E(X)$.
6. Compute the mode of $B(7, \frac{1}{4})$.
7. If X_1 and X_2 are two independent random variables following Bernoulli distribution with parameter p , then show that $X_1 + X_2$ follows binomial distribution $B(2, p)$
8. State and prove the lack of memory property of the exponential distribution.
9. Compute the m.g.f. of Uniform distribution over $(0, 2)$.
10. Obtain the points of inflexion of the normal curve $N(\mu, \sigma)$.
11. State the Lindberg-Levy form of Central limit theorem.
12. State the assumptions in the Lindberg-Levy form of Central limit theorem.



Part B

II. Answer any Six questions. Each question carries 5 marks

(6x5=30)

13. Distinguish between raw moments and central moments. Obtain the general expression for the r -th central moment in terms of raw moments.
14. Find $E(X)$, if $f(x) = \frac{e^{-1}}{x!}$, $x = 0, 1, 2, 3, \dots$ is the p.d.f of X .
15. Find the moment generating function of a random variable X whose p.d.f is $f(x) = a^x b$; $x = 0, 1, 2, \dots$, where $a+b=1$. Hence find $V(X)$.
16. Let X and Y be independent random variables such that $P(X=r) = P(Y=r) = q^r p$, $r = 0, 1, 2, \dots$ where p and q are positive numbers such that $p + q = 1$. Find (1) the distribution of $X+Y$ (2) the conditional distribution of X given $X+Y = 3$.
17. Obtain the mgf of the binomial distribution and deduce its additive property of independent binomial variates.
18. If X is a random variable distributed as $N(0,1)$ then show that X^2 has gamma distribution with parameters $m=1/2$ and $p=1/2$
19. For a rectangular distribution $f(x) = k$, $1 \leq x \leq 2$, show that $A.M > G.M > H.M$
20. A random sample of size n was taken from a population with mean μ and Standard deviation σ . Find the distribution of the sample mean $\bar{X}$ for large n .

21. How many trials should be performed so that the probability of obtaining at least 40 successes is at least 0.95, if the trials are independent and probability of success in a single trial is 0.2?

Part C

III. Answer any Two questions. Each question carries 15 marks

(2x15=30)

22. (a) Define conditional expectation and conditional variance. (b) If $f(x,y) = x+y$; $0 < x < 1$, $0 < y < 1$ is the joint p.d.f. of (X,Y) , calculate correlation between X and Y .

23. (1) For the Poisson distribution with mean m show that $\beta_1 = \frac{1}{m}$ and $\beta_2 = 3 + \frac{1}{m}$.

(2) Fit a Poisson distribution to the following data and calculate the theoretical frequencies.

x	0	1	2	3	4
f	122	60	15	2	1

24. (a) Derive the expression for the even ordered central moments of the normal distribution $N(\mu, \sigma)$. (b) find the probability that the number of heads lie in the range 185 and 220 when a fair coin is tossed 400 times.

25. For the random variable X following geometric distribution $f(x) = \frac{1}{2^x}$, $x = 1, 2, 3, \dots$. Show that $P(|x-2| \leq 2) = \frac{15}{16}$.

Find a lower bound of this probability using Tchebychev's inequality.

