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BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, NOVEMBER 2024 2018, 2019, 2020, 2021, 2022 ADMISSIONS SUPPLEMENTARY SEMESTER III - CORE COURSE (MATHEMATICS) AND (A) MT3B03B18 - Calculus

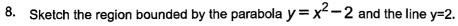
Time: 3 Hours Maximum Marks: 80

Part A

I. Answer any Ten questions. Each question carries 2 marks

(10x2=20)

- 1. Show that the curve y=sinx is concave down in the interval $(0,\pi)$.
- 2. Find the nth derivative of e^{3x} .
- 3. State the conditions to check the concavity and identify the points of inflection of a curve y=f(x).
- 4. Find the value of $\frac{\partial f}{\partial x}$ at (2,-1) if $f(x,y)=3x^3y+4xy^2-2x+4y-5$.
- 5. Use Chain rule to find the derivative $\dfrac{dw}{dt}$ at $t=\pi$, where $w=x^2+y^2$, given that x=cost, y=sint
- 6. Evaluate $\frac{\partial f}{\partial y}$ where, $f(x,y,z) = 1 + xy^2 2z^2$.
- 7. Compute the length of the curve $y=\int_{-2}^x \sqrt{3t^4-1}dt, -2\leqslant x\leqslant -1.$



- 9. Sketch the region enclosed between the curves $y = x^2$ and y = x + 6.
- 10. State First Form of Fubini's Theorem.
- 11. $\int_0^2 \int_0^1 \int_0^{1-y} dz dy dx$
- 12. $\int_0^2 \int_{y-2}^0 dx dy$ Write an equivalent double integral with order of integration reversed

Part B

II. Answer any Six questions. Each question carries 5 marks

(6x5=30)

13. If
$$y = e^{-x}(Ax + B)$$
, prove that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$.

14.
$$_{\text{lf}}y=a(1-cost),x=a(t+sint),_{\text{find}}\frac{d^2y}{dx^2}.$$

15. Show that if
$$y=sin(msin^{-1}x)$$
, then prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2y=0$.

16. Show that
$$f(x,y)=e^{-2y}cos2x$$
, satisfy Laplce's equation.

17. Find the second partial derivatives of
$$f(x, y) = x^2y + cosy + ysinx$$
.

- 18. A pyramid 3 meter high has a square base that is 3 meter on a side. The cross-section of the pyramid perpendicular to the altitude x meter down from the vertex is a square x meter on a side. Compute the volume of the pyramid.
- 19. Compute the length of the curve $x=rac{y^{rac{3}{2}}}{3}-y^{rac{1}{2}}$ from y=1 to y=9.
- 20. $\int_0^1 \int_2^{4-2r} dy dx$ Sketch the region of integration for the integral $\int_0^1 \int_2^{4-2r} dy dx$ and write an equivalent integral with order of integration reversed. Also evaluate the integral.
- 21. Compute the area of the region common to the interiors of the cardioids $r = 1 + \cos\theta$ and $r = 1 \cos\theta$

Part C

III. Answer any Two questions. Each question carries 15 marks

(2x15=30)

22. a) Find
$$y_n(0)$$
 if $y = log(x + \sqrt{1 + x^2})$

$$y = xlog \frac{x-1}{x+1}, \text{ show that } \frac{\mathrm{d}^n y}{\mathrm{d} x^n} = (-1^n)(n-2)! \left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n} \right]$$

- 23. a) Find positive numbers x, y, z such that x+y+z=18 and xyz is a maximum.
 - b) Find all the local maxima, local minima and saddle points of

$$f(x,y) = x^2 + 3xy + 3y^2 - 6x + 3y - 6$$

- 24. Compute the volume of the solid generated by revolving the triangular region bounded by the lines y=2x, y=0 and x=1 about (a) the line x=1 (b) the line x=2 (c) y axis
- 25. Let D be the region bounded by the paraboloids $z=8-x^2-y^2$ and $z=x^2+y^2$. Compute the volume of D.

