TB243677B

Reg. No :.....

Name :.....

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, NOVEMBER 2024

2018, 2019, 2020, 2021, 2022 ADMISSIONS SUPPLEMENTARY

SEMESTER III - CORE COURSE (MATHEMATICS AND CA)

MT3B03B18 - Calculus

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Show that the curve y=sinx is concave down in the interval $(0, \pi)$.
2. Find the nth derivative of e^{3x}.
3. State the conditions to check the concavity and identify the points of inflection of a curve y=f(x).
4. Find the value of $\frac{\partial f}{\partial x}$ at (2,-1) if $f(x,y) = 3x^3y + 4xy^2 - 2x + 4y - 5$.
5. Use Chain rule to find the derivative $\frac{dw}{dt}$ at $t = \pi$, where $w = x^2 + y^2$, given that $x = cost, y = sint$.
6. Evaluate $\frac{\partial f}{\partial y}$ where, $f(x,y,z) = 1 + xy^2 - 2z^2$.
7. Compute the length of the curve $y = \int_{-2}^{x} \sqrt{3t^4 - 1}dt, -2 \leqslant x \leqslant -1$.
8. Sketch the region bounded by the parabola $y = x^2 - 2$ and the line y=2.
9. Sketch the region enclosed between the curves $y = x^2$ *and* $y = x + 6$.
10. State First Form of Fubini's Theorem.
11. Compute $\int_0^2 \int_0^1 \int_0^{1-y} dz\,dy\,dx$.
12. Write an equivalent double integral with order of integration reversed $\int_0^2 \int_{y-2}^{0} dx\,dy$

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. If $y = e^{-x}(Ax + B)$, prove that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$.
14. If $y = a(1 - cost), x = a(t + sint)$, find $\frac{d^2y}{dx^2}$.
15. Show that if $y = sin(msin^{-1}x)$, then prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$.
16. Show that $f(x,y) = e^{-2y}cos2x$, satisfy Laplce's equation.
17. Find the second partial derivatives of $f(x,y) = x^2y + cosy + ysinx$.

18. A pyramid 3 meter high has a square base that is 3 meter on a side. The cross-section of the pyramid perpendicular to the altitude x meter down from the vertex is a square x meter on a side. Compute the volume of the pyramid.

19. Compute the length of the curve $x = \frac{y^{\frac{3}{2}}}{3} - y^{\frac{1}{2}}$ from y=1 to y=9.

20. Sketch the region of integration for the integral $\int_0^1 \int_2^{4-2x} dydx$ and write an equivalent integral with order of integration reversed. Also eveluate the integral.

21. Compute the area of the region common to the interiors of the cardioids $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. a) Find $y_n(0)$ if $y = log(x + \sqrt{1+x^2})$

b) If $y = xlog\frac{x-1}{x+1}$, show that $\frac{d^n y}{dx^n} = (-1^n)(n-2)!\left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$

23. a) Find positive numbers x, y, z such that $x + y + z = 18$ and xyz is a maximum.

b) Find all the local maxima, local minima and saddle points of

$f(x,y) = x^2 + 3xy + 3y^2 - 6x + 3y - 6$.

24. Compute the volume of the solid generated by revolving the triangular region bounded by the lines y=2x, y=0 and x=1 about (a) the line x=1 (b) the line x=2 (c) y axis

25. Let D be the region bounded by the paraboloids $z=8-x^2-y^2$ and $z=x^2+y^2$. Compute the volume of D.