TB242101M

Reg. No :.....

Name :.....

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, MARCH 2024

2023 ADMISSIONS REGULAR

SEMESTER II - COMPLEMENTARY COURSE 1

MT2B02B23 - Exponential, Logarithmic Functions, Linear Algebra and Advanced Calculus

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Write the exponential equation to find the growth rate.
2. $(a) 4\ ln\frac{1}{3}$

   Simplify : $(b) ln\ x^7 - ln\ x^2$
3. $(a) ln12 + ln3 - ln4$

   Simplify : $(b) \frac{e^{2x}}{e^{8x}}$
4. Define Square Marix.
5. Find the determinant of the matrix $\begin{bmatrix} 2 & 5 & 9 \\ 16 & 4 & 20 \\ 28 & 7 & 35 \end{bmatrix}$.
6. Find AB and $A^T B^T$, given $A = \begin{bmatrix} 5 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 21 \\ 10 \end{bmatrix}$.
7. Define the optimal solution of a Linear programming problem.
8. Explain an optimization problem.
9. Does the linear programming problem with technical constraints 2x + 3y < 8 , x + 2y < 5 where x and y are nonnegative has a feasible solution? Give reasons.
10. State Young's Theorem for partial derivatives.
11. Using the product rule , find the first order partial derivatives of $z = (8x + 15y)(12x - 7y)$
12. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = 12x^4 - 10x^2y^3 + 15y^6$

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. How many years t will it take a sum of money P to double at 6 percent interest compounded annually?
14. A village's arable land L is eroding at a rate of 1.8 percent a year. If conditions continue, how much will be left in 15 years?
15. A ski shop discounts all its skis, poles, and bindings by 25 percent at the end of the season. Assuming that V_1 is the value of stock in its three branches prior to the discount, find the value V_2 after discount when

$$V_1 = \begin{bmatrix} 8400 & 7200 & 6800 \\ 4600 & 5400 & 5600 \\ 6200 & 7800 & 7400 \end{bmatrix}.$$

16. Use Cramer's rule to solve for the equillibrium level of income $\bar{Y}$ and the interest rate $\bar{i}$ given,
$IS: \ 0.3Y + 100i - 252 = 0$
$LM: 0.25Y - 200i - 193 = 0.$

17. A manufacturer makes two products A and B. The first requires 6 hours for processing, 4 hours for assembling, and 5 hours for packaging. The second requires 3 hours for processing, 10 hours for assembling, and 8 hours for packaging. The plant has 40 hours available for processing, 62 for assembling, and 50 for packaging. The profit margin for A is $8; for B it is $32. Express the data in equations and inequalities necessary to determine the output mix that will maximize profits.

18. A jeweler makes necklaces A and bracelets B. Necklaces have a profit margin of $33; bracelets $28. Necklaces take 3 hours for stonecutting, 8 hours for setting, and 7 hours for polishing. Bracelets take 5 hours for stonecutting, 7 hours for setting, and 3 hours for polishing. The jeweler has 40 hours for stonecutting, 70 hours for setting, and 48 hours for polishing. Convert the data to equations and inequalities needed to find the profit-maximizing output mix.

19. Explain the second-order conditions to check the nature of a critical point of a function of two variables.

20. Find all second order partial derivatives of the function $z = (5x + 12y)^4$

21. Find the first order partial derivatives of the function $z = \dfrac{5x^3 - 9y^2}{4x + 3y}$

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. (a) Differentiate $y = ln(8x^2 + 3)$

(b) Given a nomianl rate of interest $r = 8$ percent find the effective rate of interest under (i) semiannual (ii)quarterly, (iii)continous compunding, given the principal amount $P = Rs.3000.$

(c) Solve $log_a 32 = \dfrac{5}{3}$

23. Use Gaussian elimination method to solve the linear systems

(a)
$$4x + 7y = 131$$
$$8x - 3y = 41$$

(b)
$$3x_1 + 6x_2 - 5x_3 = -5$$
$$4x_1 - 7x_2 + 2x_3 = -6$$
$$-x_1 + 8x_2 + 9x_3 = 93$$

24. Use graphs to solve the following linear programming problem:

Minimize : $c = 20y_1 + 15y_2$

subject to the constraints :

$3y_1 + 2y_2 \geq 36; \quad 6y_1 + 6y_2 \geq 84; \quad y_1 + 4y_2 \geq 16; \quad y_1, y_2 \geq 0$

25. Find the output mix that will maximize profits for a firm with a maximum joint output capacity of 22 when $\pi = 160x - 4x^2 - xy - 3y^2 + 210y$