TB242542U

Reg. No :.....

Name :.....

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, MARCH 2024

2023 ADMISSIONS REGULAR

SEMESTER II - CORE COURSE MATHEMATICS

MT2C02B23 - Number Theory, Cryptography, Laplace Transforms and Conic Sections

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Prove that if $a \equiv b(mod\ n)\ then\ ac \equiv bc\ (mod\ n)$ where n > 1 is a fixed integer and a, b and c are arbitrary integers.
2. Determine the number of positive integers less than n and relatively prime to n where n = 42.
3. Compute the number of positive divisors of 180.
4. Define the terms (a) Enciphering (b) Deciphering in Cryptography.
5. Check whether the sequence 5, 13, 25, 42,90 is superincreasing.
6. Encrypt the phrase "Seize the day" using the Caesar cipher.
7. Find $L(e^{-t}\sinh 5t)$
8. Find $L^{-1}\dfrac{s^4 - 3s^2 + 12}{s^5}$
9. Find the Laplace transform of e^{3a-2bt}
10. Identify the conic section represented by the equation $y^2 - 4y - 8x - 12 = 0$
11. Write the equivalent Cartesian equation for the polarequation $r^2 = 4r\cos\theta$.
12. Find the focus and directrix of the parabola $x^2 = 6y$. Also sketch the parabola.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Calculate the remainder when the sum 1! + 2! +....+100! is divided by 12.
14. Calculate the remainder when 5^{16} is divided by 13.
15. Write a short note on Caesar cipher.
16. Encrypt the message "MATHEMATICS IS COOL" using the linear cipher C ≡ 3P + 9 (mod 26).
17. Derive the Laplace Transform of cos 2t using the definition of Laplace transform.
18. Using the method of convolution, find the inverse Laplace transform of $H(s) = \dfrac{1}{(s^2+1)^2}$
19. Calculate the inverse Laplace transform of $F(s) = \dfrac{10}{s^3 - \pi s^2}$ using integration.
20. Sketch the sets of points whose polar coordinates satisfy the conditions
    a) $1 \le r \le 2$ and $0 \le \theta \le \dfrac{\pi}{2}$
    b) $-3 \le r \le 2$ and $\theta = \dfrac{\pi}{4}$
21. Graph the curve $r^2 = 4\cos\theta$.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. (a). State and prove Wilson's theorem. (b). Find the remainder when 2(26!) is divided by 29.

23. (a) Solve the superincreasing Knapsack problem $3x_1 + 5x_2 + 9x_3 + 18x_4 + 37x_5 = 51$

(b)Use Hill's Cipher $C_1 \equiv 5P_1 + 2P_2 \ (mod\ 26)$
$C_2 \equiv 3P_1 + 4P_2 \ (mod\ 26)$ to encipher the message "HIGHLY CONFIDENTIAL".

24. a) Solve the initial value problem $y'' - y' - 6y = 0, y(0) = 0, y'(0) = 13$

b) Solve the Volterra integral equation of the second kind

$$y(t) - \int_0^t (1+\tau)y(t-\tau)d\tau = 1 - \sinh t$$

25. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(a) The ellipse is shifted 4 units to the right and 3 units up to generate a new ellipse. Find the center, foci, vertices and of the new ellipse. Also plot the new foci, vertices, and center and sketch in the new ellipse.

(b) Find the polar equation for an ellipse with semi major axis 39.44AU(astronomical unit) and eccentricity 0.25. This is the approximate size of Pluto's orbit around the sun. Also find point of closest approach and the point of farthest approach of Pluto from the sun.