TB242347C

Reg. No :.....

Name :.....

BACHELOR'S DEGREE (C.B.C.S) EXAMINATION, MARCH 2024

2023 ADMISSIONS REGULAR

SEMESTER II - CHEMISTRY COMPLEMENTARY COURSE 1

MT2B01B23 - Partial Derivatives, Multiple Integrals Trigonometry and Matrices

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Evaluate $\int_0^1 \int_0^x (3 - x - y)\, dydx$
2. Evaluate $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2)\, dzdydx$
3. Find the area of the region bounded by $y = x$ and $y = x^2$ in the first quadrant.
4. Prove that $cosh\ 3x = 4cosh^3\ x - 3\ cosh\ x.$
5. Separate into real and imaginary parts of $sinh\ (\alpha - i\beta).$
6. Show that $tanh\ (x + y) = \dfrac{tanh\ x + tanh\ y}{1 + tanh\ x \tanh\ y}$
7. Write the two dimensional Laplace equation
8. Find $\dfrac{\partial^2 f}{\partial x^2}$ of $f(x, y) = sin\ xy$
9. Find f_{xx} of $f(x, y) = xe^y + y + 1.$
10. Define rank of a matrix.
11. Find the rank of the matrix $\begin{bmatrix} 2 & 4 & 3 & 2 \\ 3 & 3 & 1 & 4 \end{bmatrix}.$
12. Find the eigen values of the matrix $\begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}.$

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Construct six different iterated triple integrals for the volume of the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 1, y = 2, z = 3.$ Evaluate one of the integrals.
14. Find the volume of the region bounded above by the elliptical paraboloid $z = 16 - x^2 - y^2$ and below by the square $R : 0 \leq x \leq 2, 0 \leq y \leq 2.$
15. Separate $log(\alpha + i\beta)$ into real and imaginary parts.
16. If $sin\ (\theta + i\ \phi) = tan\ (x + iy),$ show that $\dfrac{tan\ \theta}{tanh\ \phi} = \dfrac{sin\ 2x}{sinh\ 2y}.$
17. Find $\dfrac{\partial z}{\partial u}, \dfrac{\partial z}{\partial v}$ at $u = 0, v = 0$ if $w = x^2 - \dfrac{y}{x}$ and $x = u - 2v - 1,\ y = 2u + v - 2.$

18. Evaluate f_{yxxyz}, if $f(x, y, z) = 1 - 2xy^2z + x^2y$.

19. Verify Caley Hamilton theroem for the matrix $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

20. Reduce to the Echelon form and find the rank of the matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 1 \\ 0 & 5 & -1 \end{bmatrix}$.

21. Using Cayley Hamilton Theorem calculate A^3, if $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. (a) Find the average value of $f(x, y) = x \cos xy$ over the rectangle $R : 0 \leq x \leq \pi,\ 0 \leq y \leq 1$.

(b) Evaluate the area bounded by the parabola $x = -y^2$ and the line $y = x + 2$ and the x axis.

(c) Find the volume of the tetrahedron cut from the first octant by a plane $6x + 3y + 2z = 6$

23. Sum to infinity the series : $1 + a\cos\theta + a^2\cos 2\theta + a^3\cos 3\theta + \ldots\ldots$ where $|a| < 1$.

24. a) Find all the second order partial derivatives of the function $f(x, y) = x^2y + \cos y + y \sin x$.

b) Show that $w = \sin(x + ct)$ satisfies the equation $\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$.

25. Solve by Cramer's rule :

$$2a + b + 5c + d = 5$$
$$a + b - 3c - 4d = -1$$
$$3a + 6b - 2c + d = 8$$
$$2a + 2b + 2c - 3d = 2$$