TB241182N

Reg. No :.....

Name :.....

BACHELOR'S DEGREE (C.B.C.S.) EXAMINATION, NOVEMBER 2023

2023 ADMISSIONS REGULAR

SEMESTER I - CORE COURSE Mathematics & C.A.

MT1C01B23 - Discrete Mathematics and Trigonometry

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Define tautology and contradiction.
2. Write the negation of : "Sally is smart and hardworking " using De- Morgan's laws.
3. Find the contrapositive and the converse of the conditional statement "The home team wins whenever it is raining"
4. Define a one-one function . Give an example.
5. Let A, B, C be sets. Show that $\overline{A \cup (B \cap C)} = (\bar{C} \cup \bar{B}) \cap \bar{A}$.
6. Let f and g be functions from R to R such that f(x) = x^2 and g(x) = x- x^2 . Write f+g and f.g
7. Define Chain. Give an example.
8. Define a transitive relation. Give a transitive relation on the set of even numbers between 20 and 30
9. List the greatest member and least member in the poset (Z^+, /).
10. Write the relation connecting hyperbolic and circular sine function
11. Calculate the value of log (-i)
12. Show that cosh (x+iy) =cosh x cosy+ i sinh x siny

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. State and prove the rule of Inference 'Modus tollens'
14. Using the laws of logic show that $\neg[p \vee (\neg p \wedge q)] \equiv \neg p \wedge \neg q$
15. Sketch the graph of the function f(n) =2n+1 from the set of integers to the set of integers.
16. Establish that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ using membership table.
17. Let S be any collection of sets. Show that the relation $\subseteq$ of set inclusion is a partial order relation on S.
18. Construct Hasse diagram for the poset ({1, 2, 4, 6, 8}, /). Also identify two incomparable elements in the poset
19. If x+iy = cosh(u+iv). Deduce that $\frac{x^2}{\cosh^2 u} + \frac{y^2}{\sinh^2 u} = 1$ and $\frac{x^2}{\cos^2 v} - \frac{y^2}{\sin^2 v} = 1$
20. Show that $\log\left(\frac{\sin(x+iy)}{\sin(x-iy)}\right) = 2i\tan^{-1}(\cot x \tanh y)$
21. If sin (A+iB) =x+iy, show that $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$ and $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. (a). Show that $\sqrt{2}$ is irrational by giving a proof by contradiction

(b). Write a direct proof for the theorem "The sum of two rational numbers is rational"

23. Let f be a function from the set A to the set B and let S and T be subsets of A. Show that

(i) $f(S \cup T) = f(S) \cup f(T)$

(ii) $f(S \cap T) \subseteq f(S) \cap f(T)$. Give an example to show that $f(S) \cap f(T) \not\subset f(S \cap T)$

24. (a). Let (A, R_1) and (B, R_2) be two posets. Define a binary relation R_3 on $A \times B$ by ((a_1, b_1), (a_2, b_2)) ϵR_3 if and only if (a_1, a_2)ϵR_1 and(b_1, b_2)ϵR_2 where a_1, a_2 ϵA and b_1, b_2 ϵB. Show that R_3 is a partial order relation on $A \times B$.

(b) Construct the Hasse diagram of $(D_{12}, /)$ and show that this is a lattice.

25. (a). If $u = log\, tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$, show that (i) cosh u = sec θ (ii). sinh u = tan θ.

(b). Calculate the sum of the infinite series $c\, sin\alpha + \frac{1}{2}c^2 sin2\alpha + \frac{1}{3}c^3 sin3\alpha + \cdots$ where $|c| < 1$