TB175370V **REG. NO:....................**

NAME:........................

B. Sc. DEGREE (C.B.C.S.S) EXAMINATION , NOVEMBER 2022
(2015, 2016 and 2017 Supplementary)
SEMESTER V - CORE COURSE (MATHEMATICS)
MT5B05B- REAL ANALYSIS -I

Time: 3 Hours **Maximum: 80 Marks**

PART A

(Answer **all** questions. Each question carries 1 mark.)

1. Give an example of an infinite set having greatest member.
2. Show that there is no rational number whose square is 2.
3. Define limit point of a set.
4. Give the definition of convergence of a sequence.
5. Define distance between two non-empty sets in a metric space.
6. Define open-sphere in any metric space (X, d)

(6x1=6 marks)

PART B

(Answer any **seven** questions. Each question carries 2 marks)

7. State order completeness in $\mathbb{R}$ in terms of infimum.
8. The set $\left\{\frac{1}{n}: n\epsilon\mathbb{N}\right\}$ is unbounded. True/False. Justify.
9. Find the infimum of the set $\left\{3+\frac{1}{n^2}: n\epsilon\mathbb{N}\right\}$.
10. Show that the set $A=\left\{1,\frac{1}{2},\frac{1}{3},\ldots\ldots\ldots\ldots\ldots\ldots,\frac{1}{n}\right\}$ is neither closed nor dense in itself.
11. Show that every open interval is an open set.
12. If M and N are neighbourhoods of a point x, then show that $M\cap N$ is also a neighbourhood of x.
13. Show that every convergent sequence is bounded.
14. Show that $\lim_{n\to\infty}\sqrt[n]{n}=1$.
15. Show that the subset $A=[0,1)$ of the metric space (X,d) where $X=[0,2)$ and d is the usual metric is an open set.
16. Find a dense subset of the metric space (R, d) where R is the set of real numbers and d is the usual metric.

(7x2= 14 marks)

P.T.O

Part C

(Answer any **five** questions. Each question carries 6 marks.)

17. State and prove Archimedean property of real numbers.
18. If $S \subseteq T \subseteq \mathbb{R}$, where $S \neq \phi$, then show that:
    i) If T is bounded abov, then $\sup S \leq \sup T$.
    ii) If T is bounded below, then $\inf T \leq \inf S$.

19. Prove that the derived set of a set is closed.
20. Prove that a set is closed if and only if its complement is open.
21. State and prove Sandwich theorem.
22. Prove that every bounded sequence with a unique limit point is convergent.
23. Explain the concepts interior of a set, exterior of a set, Frontier and boundary of a set in any metric space(X, d) with the help of examples.
24. In any metric space(X, d), prove that:
    i) The intersection of an arbitrary family of closed sets is closed.
    ii) The union of a finite number of closed sets is closed.

(5x6= 30 marks)

PART D

(Answer any **two** questions. Each question carries 15 marks).

25. Prove the equivalence of Dedekind's property and order completeness property of real numbers
26. State and prove Bolzanno Weirstrass theorem for sets.
27. State and prove Cauchy's general principle of convergence of sequences.
28. Let (X, d) be a metric space and $Y \subseteq X$, then show that a subset A of Y is open in (y, d_Y) if and only if there exists a set G open in (X, d) such that $A = G \cap Y$.

(2x15= 30 marks)