TB145800 V **Reg No:.............................**

Name:................................

B. Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2022

(2014 Admission Supplementary)

FIFTH SEMESTER - CORE COURSE (MATHEMATICS)

Common to B.Sc. MATHEMATICS and COMPUTER APPLICATIONS

MAT5MA: MATHEMATICAL ANALYSIS

Time: 3 hours **Maximum marks: 80**

PART A

Short answer questions

(Answer all questions. Each question carries 1 mark)

1. State Cauchy's first theorem on limits.
2. Define Infimum of a set .
3. Give an example of a bounded set having no limit point.
4. State Bolzano Weierstrass theorem for sets.
5. Define an open set .
6. Give an example to show that every bounded sequence need not be convergent.
7. Find the limit point of the set {2, 4, 6}
8. Find $\lim_{n\to\infty} \frac{(3n+1))}{(n+3)}$.
9. Show that Im (iz) = Re z.
10. Express (1+ i) in the exponential form

$(\mathbf{10 \times 1 = 10})$

PART B

(Answer any 8 questions. Each question carries 2 marks)

11. Find the interior of the set of rational numbers.
12. Prove that the greatest number of a set if it exists is the supremum of the set.
13. Construct $\{a_n\}$ such that $a_n = n + 1\ \forall n \epsilon \mathbb{N}$.
14. Give an example of a bounded set having no limit point and a bounded set having infinite number of limit points.
15. Show that a set cannot have more than one supremum.
16. Show that the interior of a finite set is the null set.
17. Prove that every convergent sequence is bounded.

18. If S and T are subsets of real numbers, then show that $S \subseteq T \Rightarrow S' \subseteq T'$.
19. Show that the set of rational numbers in [0,1] is countable.
20. Show that every monotonic decreasing sequence which is bounded below, converges to the infimum
21. Sketch the set $|2z + 3| > 4$.
22. Prove that $|z_1 + z_2|^2 \leq |z_1|^2 + |z_2|^2$.

$(\mathbf{8 \times 2 = 16})$

PART C

(Answer any 6 questions. Each question carries 4 marks)

23. Show that derived set of a bounded set is bounded.
24. State and prove Archimedean property.
25. Prove that the interior of a set S is an open subset of S.
26. If M and N are neighbourhoods of a point x, then show that M∩N is also a neighbourhood of x.
27. Show that the intersection of an arbitrary family of closed sets is closed.
28. Prove that a set is closed if its complement is open.
29. Prove that the Cartesian product of two countable sets is countable.
30. Show that every bounded sequence with a unique limit point is convergent.
31. Use De-Moivre's formula to derive $\sin 3\theta = 3\cos^2\theta \sin\theta - \sin^3\theta$.

$(\mathbf{6 \times 4 = 24})$

PART D

Answer any 2 questions. Each question carries 15 marks.

32. State and prove the equivalence of order completeness property and Dedekind's property of $\boldsymbol{R}$.
33. State and prove Bolzano Weierstrass's theorem for sets.
34. State and Prove Cauchy's Second theorem on limits.
35. Prove that a necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded.

$(2 \times 15 = 30)$