TB205370V

Reg. No :...............................

Name :..................................

B. Sc.DEGREE (C.B.C.S.) EXAMINATION, NOVEMBER 2022

2020 ADMISSIONS REGULAR AND 2019, 2018 ADMISSIONS SUPPLEMENTARY

SEMESTER V - CORE COURSE (MATHEMATICS)

MT5B06B18 - REAL ANALYSIS I

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Prove or disprove : A bounded set always has the infimum and the supremum.
2. Let M be the infimum of a set S and 'a' be a real number greater than M. Can 'a' be an upper bound of S. Why?
3. Define a perfect set. Give an example of a bounded set which is perfect.
4. Explain that the set {1,4,9,16,…………} is countable.
5. Find all adherent points of the set {4, 6} ?
6. Obtain the derived set of the following sets :
   (a) (8, 9)
   $(b)\left\{1, -1, 1\frac{1}{2}, -1\frac{1}{2}, 1\frac{1}{3}, -1\frac{1}{3}, ...\right\}$
7. Show that every Cauchy sequence is bounded.
8. Define an "oscillate infinitely sequence" . Give an example.
9. Find the limit superior and limit inferior of the sequence $\{(0.5)^n\}$.
10. Show that the sequence $\{n^3+1\}$ diverges to +∞ .
11. Define a bounded metric space. Give an example.
12. Check whether the function $d : R \times R \to R$ defined by d(x, y) = min {xy, 0} is a metric on R where R is the set of real numbers.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Prove that N is order complete where N is the set of natural numbers.
14. If a and b are any two positive real numbers then prove that there exist a positive integer n such that na > b.
15. Prove that closure of a bounded set is bounded.
16. Prove that the infimum of a bounded set is always a member of its closure.
17. State and Prove Cesaro's theorem.
18. Show that the sequence $\{(a_n)^{1/n}\}$ converges and find its limit where $a_n = \frac{(3n)!}{(n!)^2}$
19. Evaluate $\lim_{n\to\infty} \frac{1}{n}[1 + 2^{\frac{1}{2}} + 3^{\frac{1}{3}} + ... + n^{\frac{1}{n}}].$
20. Prove or disprove : In any metric space (X, d), the intersection of an arbitrary family of open sets is open.
21. Is (Q, d) where d is the usual metric a complete metric space ? justify your answer.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. Explain the proof which shows that the order completeness property of real numbers is implied and is implied by its Dedekind's form of completeness property.
23. Prove that (i) The set of real numbers in [0,1] is uncountable. (ii) The Cartesian product of two countable sets is countable.
24. State and prove Cantor’s intersection theorem for real line.
25. Define a perfect set and prove that Cantor set is a perfect set.