TB205375V **Reg. No :..............................**

Name :..............................

B. Sc. DEGREE (C.B.C.S.) EXAMINATION, NOVEMBER 2022

2020 ADMISSIONS REGULAR AND 2019, 2018 ADMISSIONS SUPPLEMENTARY

SEMESTER V - CORE COURSE (MATHEMATICS)

MT5B08B18 - ABSTRACT ALGEBRA

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Give an example of an associative binary operation that is not commutative.
2. Examine whether the set of all $n \times n$ diagonal matrices form a group under matrix multiplication.
3. Define the orbits of a permutation.
4. Define cycle.
5. Compute the gcd of 58 and 68.
6. Evaluate all the orbits of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 3 & 6 & 2 & 4 \end{pmatrix}$
7. Define the kernel of a homomorphism.
8. Define inner automorphism of a group G.
9. Find $\phi(18)$ of the group homomorphism $\phi : \mathbb{Z} \mapsto \mathbb{Z}_{10}$ such that $\phi(1) = 6$
10. Calculate the kernel of the homomorphism $\phi : \mathbb{Z} \mapsto \mathbb{Z}_7$ such that $\phi(1) = 4$
11. Define a field. Give an example.
12. Solve the equation $3x = 2$ in the field $\mathbb{Z}_7$.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Establish the uniqueness of identity element in a group.
14. Show that if $(a * b)^2 = a^2 * b^2$ for a and b in a group G, then $a * b = b * a$.
15. Show that every group of prime order is cyclic.
16. Let A be a nonempty set and let S_A be the collection of all permutations of A. Deduce that S_A is a group under permutation multiplication.
17. Let H be a subgroup of a group G. Let the relation be $\tilde{}_L$ be defined on G by $a \tilde{}_L b$ if and only if $a^{-1}b \in H$. Show that $\tilde{}_L$ is an equivalence relation on G.
18. Let ϕ be a homomorphism of a group G into a group G'. If K' is a subgroup of $G' \cap \phi[G]$, then prove that $\phi^{-1}[K']$ is a subgroup of G.
19. Compute all the cosets of the subgroup $< 4 >$ of $\mathbb{Z}_{12}$
20. Deduce that the cancellation laws hold in a ring R if and only if R has no zero divisors.
21. Show that if a and b are nilpotent elements of a commutative ring, then $a + b$ is also nilpotent.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. Determine whether the given set of matrices is a group.

    a) All $n \times n$ upper triangular matrices under matrix addition

b) All $n \times n$ upper triangular matrices under matrix multiplication

c) All $n \times n$ upper triangular matrices with determinant 1 under matrix multiplication

23. Establish that the subgroups of $\mathbb{Z}$ under addition are precisely the groups $n\mathbb{Z}$ under addition for $n \in \mathbb{Z}$
24. Let G be a cyclic group with generator a. Establish that

    a) If the order of G is infinite, then G is isomorphic to $(\mathbb{Z}, +)$.

    b) If G has finite order n, then G is isomorphic to $(\mathbb{Z}_n, +_n)$.
25. a) State Lagrange's theorem and construct the proof of Lagrange's theorem b) Establish that every group of prime order is cyclic. c) Prove that the order of an element of a finite group divides the order of the group.