TB213500V **Reg. No :...............................**

Name :..................................

B. Sc. DEGREE (C.B.C.S) EXAMINATION, NOVEMBER 2022

(2021 Admissions Regular,2020 Admissions Supplementary/Improvement,2019 & 2018 Admissions Supplementary)

SEMESTER III - COMPLEMENTARY COURSE 1 (STATISTICS)

ST3C01B18 - PROBABILITY DISTRIBUTIONS

(For Maths and Physics)

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. If the moment generating function of a random variable X is $(1 - t)^{-1}$, find E(X).
2. Define expectation of a function of a random variable.
3. Define mathematical expectation.
4. Write any two properties of Moment generating function of a random variable.
5. If X ~ P(λ), find E(X).
6. If X and Y are independent Poisson variates with means 2 and 3 respectively find the mean and variance of 2X+ 3Y.
7. Compute the mode of B(7, ¼).
8. Define Beta distribution of the second type.
9. State any four properties of Normal distribution.
10. If X follows Uniform distribution over [0,1], then state the distribution Y = - 2 log X.
11. If X is a random variable with E(X) = 3 and V(X) = 2, find h so that P[|X – 3 |< h] ≥ 0.99
12. A symmetric die is thrown 600 times. Find a lower bound for the probability of getting 80 to 120 sixes.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. State and prove Cauchy-Schwartz inequality.
14. If the joint pdf of a random variable (X,Y) is f(x,y) = x + y; 0 < x < 1, 0 < y < 1. Find covariance between X and Y.
15. The joint p.d.f. of (X,Y) is f(x,y) = $\frac{x+y}{21}$; x=1,2,3 , y=1,2.Find E(X|Y=2)
16. Let X and Y be independent random variables such that P(X= r) = P(Y= r) = $q^r p$, r = 0, 1, 2,. …. where p and q are positive numbers such that p + q = 1. Find (1) the distribution of X+Y (2) the conditional distribution of X given X+Y = 3.
17. If X and Y follow geometric distributions with parameter `p', find (i) the distribution of Z = X + Y (ii) P(X = Y).
18. If X is a random variable distributed as N(0,1) then show that X^2 has gamma distribution with parameters m=1/2 and p= 1/2
19. Obtain the points of inflexion of the normal curve with mean μ and Standard deviation σ.
20. State and prove Tchebychev's inequality.
21. State central limit theorem for identically distributed random variables. Deduce the normal approximation of the binomial by using central limit theorem.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. (a)Define conditional expectation and conditional variance.(b) If f(x,y)= x+y; 0<x<1, 0<y<1 is the joint p.d.f. of (X,Y), calculate correlation between X and Y.

23. (1) For the Poisson distribution with mean m show that $\beta_1 = \frac{1}{m}$ and $\beta_2 = 3 + \frac{1}{m}$.

    (2) Fit a Poisson distribution to the following data and calculate the theoretical frequencies.

x	0	1	2	3	4
f	122	60	15	2	1

24. Fit a Normal distribution to the following data and find the expected frequencies

Class intervals	21-24	25-28	29-32	33-36	37-40
Frequencies	4	8	12	10	6

25. A random sample of size 100 is taken from an infinite population with mean 75 and variance 256
    (a) Using Tchebychev's inequality, find P[67 < $\bar{X}$ < 83]
    (b) Using Central limit theorem, find P[67 < $\bar{X}$ < 83]