TB213440V

Reg. No :...............................

Name :.................................

B. Sc. DEGREE (C.B.C.S) EXAMINATION, NOVEMBER 2022

(2021 Admissions Regular,2020 Admissions Supplementary/Improvement,2019 & 2018 Admissions Supplementary)

SEMESTER III - COMPLEMENTARY COURSE 1(MATHEMATICS)

MT3C01B18 - VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND ANALYTIC GEOMETRY

(For Chemistry and Physics)

Time : 3 Hours **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. Explain product rule and quotient rule for gradient functions.
2. Find the direction in which the function $f(x,y,z)=\left(\frac{x}{y}\right)-yz$ decrease most rapidly at the point $(4,1,1)$.
3. Find the directions in which the function $f(x,y)=x^2+xy+y^2$ increase rapidly at the point $(-1,1)$.
4. Explain exact differential form with an example.
5. Evalauate the flux of the field $\boldsymbol{F}=-y\boldsymbol{i}+x\boldsymbol{j}$ across the curve $\boldsymbol{r}(t)=(\cos t)\,\boldsymbol{i}-(6\sin t)\,\boldsymbol{j}$.
6. Evaluate $\int_C (x+y)\,ds$ where C is the straight -line segment joining $x=t,\ y=(1-t),\ z=0$ from $(0,1,0)$ to $(1,0,0)$.
7. Write the general form of Lagrange's Equation.
8. Solve the differential equation $x^2dy+y^2dx=0$.
9. Examine whether the equation $(e^y+1)\cos x\, dx + e^y \sin x\, dy = 0$ is exact or not.
10. List all Polar Coordinate representations for the point (-3, 0).
11. List all polar coordinate representatives for the point $P\left(2,\frac{\pi}{6}\right)$.
12. Identify the focus and directrix of the parabola $y^2= 10x$.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. (a) Estimate the gradient of the function $f(x,y,z)=\frac{x^2}{2}+\frac{y^2}{2}$ at the point $(1,1)$

    (b) Find the equation of a tangent line to the curve $x^2-y=1$ at the point $(\sqrt{2},1)$.
14. Express acceleration $\boldsymbol{a}$ in the form $\boldsymbol{a}=a_T\boldsymbol{T}+a_N\boldsymbol{N}$ where $\boldsymbol{r}(t)=t^2\,\boldsymbol{i}+(t+\frac{1}{3}t^3)\,\boldsymbol{j}+(t-\frac{1}{3}t^3)\,\boldsymbol{k}$ at $t=0$.
15. Find a potential function f for the field $\boldsymbol{F}=(y+z)\,\boldsymbol{i}+(x+z)\,\boldsymbol{j}+(x+y)\boldsymbol{k}$.
16. Apply Stoke's Theorem to calculate the counterclockwise circulation of the field $\boldsymbol{F}=(x^2-y)\boldsymbol{i}+4z\boldsymbol{j}+x^2\boldsymbol{k}$ around the curve C in which the plane z=2 meets the cone $\sqrt{x^2+y^2}=z$ as viewed from above.
17. Determine whether the differential equation $2xy\,dx+(y^2+x^2)dy=0$ is exact and hence solve.
18. Solve the differential equation $(px - y)(x - py)= 2p$
19. Write the Polar Equation for the circle whose Cartesian Equation is given by $(x-6)^2+y^2=36$. Also sketch the circle.

20. By changing to Cartesian coordinates show that $r = 8\sin\theta$ is a circle and $r = \dfrac{2}{1-\cos\theta}$ is a parabola.

21. Determine the eccentricity of the hyperbola $12x^2-27y^2= 108$. Also sketch the hyperbola and label its center, vertices, asymptotes and focii.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. (a) Find κ and τ for the curve $\boldsymbol{r}(t) = (1+3t)\,\boldsymbol{i} + (t-2)\,\boldsymbol{j} - 3t\,\boldsymbol{k}$

    (b) Find the tangential and normal components of acceleration for the curve $\boldsymbol{r}(t) = (\cos t)\,\boldsymbol{i} + (\sin t)\,\boldsymbol{j} + t\,\boldsymbol{k}$.

23. Establish that the conclusion of both forms of Green's Theorem are true by evaluating both sides of the equations for the field $\boldsymbol{F} = -y\boldsymbol{i} + x\boldsymbol{j}$, where the domain of integration is the disk $R\colon x^2+y^2 \le a^2$ and its bounding circle $C : \boldsymbol{r} = (a\cos t)\boldsymbol{i} + (a\sin t)\boldsymbol{j},\ 0 \le t \le 2\pi$.

24. (a) Determine the general solution of $\left(\dfrac{dy}{dx}\right)^3 = \dfrac{dy}{dx}e^{2x}$ by solving for p.

    (b). Solve the homogeneous differential equation $(y^2+yx)dx+x^2dy =0$.

25. (a). Find the standard form equation of the ellipse having one focus at (4, 0) and $x = \dfrac{16}{3}$ as the corresponding directrix.

    (b) Find an equation for the hyperbola with eccentricity $\dfrac{3}{2}$ and directrix x = 2.