TB213430V

Reg. No :...............................

Name :..................................

B. Sc. DEGREE (C.B.C.S) EXAMINATION, NOVEMBER 2022

(2021 Admissions Regular,2020 Admissions Supplementary/Improvement,2019 & 2018 Admissions Supplementary)

SEMESTER III - CORE COURSE (MATHEMATICS)

(For Maths & Computer Applications)

MT3B03B18 - CALCULUS

Time : 3 Hours

Maximum Marks : 80

Part A

I. Answer any Ten questions. Each question carries 2 marks **(10x2=20)**

1. If $y = Acos2x + Bsin2x$, then prove that $\frac{d^2y}{dx^2} + 4y = 0.$
2. State the conditions to check the concavity and identify the points of inflection of a curve y=f(x).
3. Expand sinx by Maclaurin's series.
4. Evaluate $\frac{dw}{dt}$, where $w = xy + z, x = cost, y = sint, z = t$.
5. If $f(x, y) = xtan^{-1}(xy)$, find $\frac{\partial f}{\partial y}$.
6. The plane x =1 intersects the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the parabola at (1,2,5).
7. Compute the length of the curve $y = \int_{-2}^{x} \sqrt{3t^4 - 1}dt, -2 \leqslant x \leqslant -1$.
8. State Pappus Theorem for Surface Areas.
9. State Pappus Theorem for volumes.
10. Define area of a closed bounded region R using a double integral.
11. Compute $\int_0^1 \int_0^2 \int_1^2 (x^2yz)dzdydx$.
12. Compute $\int_0^2 \int_{-\pi}^0 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\rho^3 sin2\phi)d\phi d\theta d\rho$.

Part B

II. Answer any Six questions. Each question carries 5 marks **(6x5=30)**

13. Use Taylor's Theorem to prove that

$\tan^{-1}(x+h) = \tan^{-1}x + h\sin z(\sin z) - (h\sin z)^2\frac{\sin 2z}{2} + (h\sin z)^3\frac{\sin 3z}{3} - ...$, where $z = cot^{-1}x$.

14. Find the nth derivative of $e^{ax}cos^2xsinx.$
15. Show that if $y = sin(msin^{-1}x)$, then prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$.
16. Find the second partial derivatives of $f(x, y) = x^2y + cosy + ysinx.$
17. If $sinz = \frac{x + y}{\sqrt{x} + \sqrt{y}}$, prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{2}tanz.$

18. The region between the curve $y = \sqrt{x}$, $0 \leq x \leq 4$ and the x axis is revolved about the x-axis to generate a solid. Calculate the volume of the solid.

19. Compute the volume of the solid generated by revolving the region bounded by $y = 4 - x^2, y = 2 - x$ about the x -axis.

20. Evaluate $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_0^{2x+y} dz dx dy$.

21. Sketch the region of integration for the integral $\int_0^1 \int_y^{\sqrt{y}} dx dy$ and write an equivalent double integral with order of integration reversed. Also evaluate the integral.

Part C

III. Answer any Two questions. Each question carries 15 marks **(2x15=30)**

22. a) Find $y_n(0)$ if $y = log(x + \sqrt{1 + x^2})$

b) If $y = xlog\frac{x-1}{x+1}$, show that $\frac{d^n y}{dx^n} = (-1^n)(n-2)! \left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$

23. a) Find the absolute maximum and minimum of $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$.

b) A rectangular box, open at the top, is to have a volume of 32 cubic feet. What must be the dimensions so that the total surface area is a minimum?

24. Compute the volume of the solid generated by revolving the triangular region bounded by the lines y=2x, y=0 and x=1 about (a) the line x=1 (b) the line x=2 (c) y axis

25. (a) Evaluate $\int_0^4 \int_{x=\frac{y}{2}}^{x=\frac{y}{2}+1} \frac{2x-y}{2} dx dy$ by applying the transformation $u = \frac{2x-y}{2}$, $v = \frac{y}{2}$ and integrating over an appropriate region in the u-v plane.

(a) Determine the spherical coordinate equation for the sphere $x^2+(y-1)^2+z^2=1$