B. Sc. DEGREE (C.B.C.S.) EXAMINATION, NOVEMBER 2022

(2022 Admissions (regular) 2021 Admissions (Improvement / Supplementary), 2020, 2019, 2018, Admissions Supplementary)

SEMESTER I - COMPLEMENTARY COURSE 1 (MATHEMATICS) (For Physics & Chemistry)

MT1C01B18 - DIFFERENTIAL AND INTEGRAL CALCULUS

Time : 3 Hours   **Maximum Marks : 80**

Part A

I. Answer any Ten questions. Each question carries 2 marks   **(10x2=20)**

1. Calculate the second derivative of $y = -x^2 + 3$

2. Calculate the second derivative of $y = x^2 + x + 8$

3. Determine the value of $(f \circ g)'$ at $x = 1$, given $f(u) = u^5 + 1, \quad u = g(x) = \sqrt{x}$

4. Write the Mean Value Theorem.

5. Write first derivative test for monotonic functions.

6. Determine the critical points of f, given $f'(x) = x^{\frac{-1}{3}}(x + 2)$

7. Write any two properties of definite integrals.

8. Solve $\frac{d}{dx}\int_a^x \cos t \, dt$

9. Given $\int_0^3 g(t) \, dt = \sqrt{2}$. Evaluate $\int_{-3}^0 \frac{g(r)}{\sqrt{2}} \, dr$.

10. Write the equation for finding the surface area for revolution about the y-axis

11. Find the volume of the solid generated by revolving the region between the Y axis and the curve $x = 2\sqrt{y}, 0 \le y \le 4$ about Y axis.

12. Evaluate the surface area generated by revolving the curve $x = \sin y \quad 0 \le y \le \pi$ about the y-axis

Part B

II. Answer any Six questions. Each question carries 5 marks   **(6x5=30)**

13. Using implicit differentiation calculate $y^2 \cos\left(\frac{1}{y}\right)) = 2x + 2y.$

14. Calculate an equation for the line tangent to the curve $x = t - \sin t, \ y = 1 - \cos t$ at the point $t = \frac{\pi}{3}$. Also, find the value of $\frac{d^2 y}{dx^2}$ at this point.

15. Compute the equation of a function with the derivative $f'(x) = 2x - 1$ whose graph passes through the point $P(0,0).$

16. Find the extreme values of the function $f(t) = t^3 - 3t^2, \ -\infty \le t \le 3$

17. Evaluate the total area between the region $y = x^3 - 4x$, $-2 \le x \le 2$, and the $x - axis$.

18.

Graph the integrand of $\int_{\frac{1}{2}}^{\frac{3}{2}}(-2x+4)dx$ and use area to evaluate the integral.

19.

Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1, x = 4$ about the line $y = 1.$

20. Find the area of the region enclosed by the curves $x + y^2 = 3$ and $4x + y^2 = 0$

21. A pyramid 3 m high has a square base that is 3 m on a side. The cross section of the pyramid perpendicular to the altitude x m down from the vertex, is a square x m on a side. Evaluate the volume of the pyramid.

Part C

III. Answer any Two questions. Each question carries 15 marks (2x15=30)

22.

(a) Evaluate $\lim\limits_{x \to 2} \dfrac{\sqrt{x^2+12}-4}{x-2}$.

(b) Deduce the equation for the line perpendicular to the tangent to the curve $y = x^3 - 4x + 1$ at the point $(2,1).$

(c) At time t, the position of a body moving is $s(t) = -t^3 + 3t^2 - 3t, 0 \le t \le 3.$ Estimate it's speed, velocity , acceleration and jerk.

23.

(a) Given the acceleration $a = 9.8,$ initial velocity $v(0) = -3,$ and the initial position $s(0) = 0$ of a body moving on a coordinate line. Find the body's position at time $t.$

(b) Determine whether the function $f(x) = \sqrt{x(1-x)}$, $x \in [0,1]$ satisfy the mean value theorem

24.

(a) Calculate $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \dfrac{1-\cos 2t}{2}$

(b) Determine the area of the region between the X-axis and the graph of the function $f(x) = x^3 - 3x^2 + 2x$,$0 \le x \le 2$

25.

(a) Determine the area of the surface generated by revolving the curve $y = x^3, 0 \le x \le \frac{1}{2}$ about the $x - axis.$

(b) The circle $x^2 + y^2 = a^2$ is rotated about the $x - axis$ to generate a sphere. Evaluate its volume.