**ST. TERESA'S COLLEGE (AUTONOMOUS)**

**ERNAKULAM**

**FINAL YEAR B.Sc PHYSICS**

**PROJECT REPORT**

**2021-22**

# STUDY OF LISSAJOUS PATTERNS USING OSCILLOSCOPE AND PENDULUM METHOD

**PROJECT REPORT**

Submitted by

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**Register No: AB19PHY002**

Under the guidance of

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Submitted to,

**Mahatma Gandhi University, Kottayam**

In partial fulfilment of the requirement for the Award of

**BACHELOR'S DEGREE OF SCIENCE IN PHYSICS**

**St. TERESA'S COLLEGE (AUTONOMOUS)**

**ERNAKULAM, KOCHI-682021**

**ST. TERESA'S COLLEGE (AUTONOMOUS)**

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# CERTIFICATE

This is to certify that the project report entitled " **STUDY OF LISSAJOUS PATTERNS USING OSCILLOSCOPE AND PENDULUM METHOD** " is a bonafide work by **Alicia Mariyam Bobby**, St. Teresa's College Ernakulam, under my supervision at the Department of Physics, St. Teresa's College, Ernakulam for the partial fulfilment of the award of Degree Of Bachelor of Science in Physics during the academic year 2020-21 The work presented in this dissertation has not been submitted for any other degree in this or any other university.

Supervising Guide

for Dr. Mary Vinaya

Head of the Department

Dr. Priya Parvathi Ameena Jose

1. Examiner 1:
2. Examiner 2:

Place: Ernakulam

Date: 9/5/2022

**ST. TERESA'S COLLEGE (AUTONOMOUS)**

**ERNAKULAM**

**B.Sc PHYSICS**

**PROJECT REPORT**

**Name: ALICIA MARIYAM BOBBY**

**Register Number: AB19PHY002**

**Year of Work: 2021-2022**

This is to certify that this project work entitled " **STUDY OF LISSAJOUS PATTERNS USING OSCILLOSCOPE AND PENDULUM METHOD** " is a bonafide work by Alicia Mariyam Bobby, St. Teresa's College Ernakulam.

**Staff Member in-charge**

for **Dr. Mary Vinaya**

**Head of the Department**

**Dr. Priya Parvathi Ameena Jose**

Submitted for the University Examination held at St. Teresa's College, Ernakulam.

Date: 9/5/2022

Examiner

## DECLARATION

I, Alicia Mariyam Bobby (Register No: AB19PHY002), final year B.Sc. Physics student, Department of Physics, St. Teresa's College, Ernakulam, do hereby declare that the project work entitled " **STUDY OF LISSAJOUS PATTERNS USING OSCILLOSCOPE AND PENDULUM METHOD** " has been originally carried out under the guidance and supervision of Dr. Mary Vinaya, Professor, Department of Physics, St. Teresa's College (Autonomous), Ernakulam, in partial fulfilment for the award of the Degree of Bachelor of Physics. I further declare that this project is not partially or wholly submitted for any other purpose and the data included in the project is collected from various sources and are true to the best of my knowledge.

Alicia Mariyam Bobby

Place: Ernakulam

Date: 9/5/2022.

# STUDY OF LISSAJOUS PATTERNS USING OSCILLOSCOPE AND PENDULUM METHODS

## *ACKNOWLEDGMENT*

*I am bringing out this project report with immense pleasure and sense of satisfaction. I would like to express our sincere gratitude to Dr. Mary Vinaya, our project in-charge, for guiding for her valuable inputs and guidance.*

*I would also like to extend my sincere thanks to all the faculty members of the Physics Department, non-teaching staff and our friends for their valuable suggestions and support Above all, I thank God Almighty for showering abundant blessings upon me to get through this project.*

*Alicia Mariyam Bobby*

**ABSTRACT**

Any of an infinite variety of curves formed by combining two mutually perpendicular simple harmonic motions, commonly exhibited by the oscilloscope, and used in studying frequency, amplitude, and phase relations of harmonic variables are called Lissajous Figures. They merge mathematical elegance, engineering applications, and artistic possibilities. Initially investigated by Nathaniel Bowditch in 1815 who created these figures using a harmonograph, its practical application and analysis was done in 1857 by Jules Antoine Lissajous (for whom they are named), a professor of mathematics at the Lycée Saint-Louis in Paris.

Here we see how Lissajous figures differ from the patterns created by a harmonograph and how they are used to assess the frequency relation between sinusoidal waves producing the patters. Although such differences can be measured numerically, the Lissajous figure makes it easy to do a visual real-time observation of the phase relationship and subtle changes in it between the left and right channels of a stereo audio signal. This is done by connecting the signal at the input of the linear channel to the scope's X-input and the output to the Y-input, the static input/output phase and amplitude relationship are visible. Any changes will be seen as well.

# CONTENTS

# CHAPTER 1

# INTRODUCTION

Lissajous patterns are formed when you combine periodic waves moving back and forth with periodic waves moving up and down. This exhibit does this electronically allowing the visitor to control the frequency of the X and Y motions independently. The resulting pattern can be observed on an oscilloscope and viewed on moving speakers. If the frequencies are high enough, the speakers produce sound. The combination of tones can be correlated with the scope pictures.

The oscilloscope can measure voltage, frequency, and phase angle. Two-channel measurements are very useful and are presented. We also can get Lissajous figures in oscilloscope other than to get the waveform figures.

In electronic applications we can generate Lissajous patterns by applying different signals to the horizontal and vertical inputs of an oscilloscope. This technique was often used to measure unknown frequencies before the invention of frequency meters. A signal of known frequency was applied to the vertical input. The resulting pattern was a function of the ratio of the two frequencies.

A Lissajous figure is produced by taking two sine waves and displaying them at right angles to each other. This can be easily done on an oscilloscope in XY mode. If the oscilloscope has the x-versus-y capability, one can apply one signal to the vertical deflection plates while applying a second signal to the horizontal deflection plates. The horizontal sweep section is automatically disengaged at this time. The resulting waveform is called Lissajous figure. This mode can be used to measure phase or frequency relationships between two signals.

## 1.1 Lissajous Figure Use

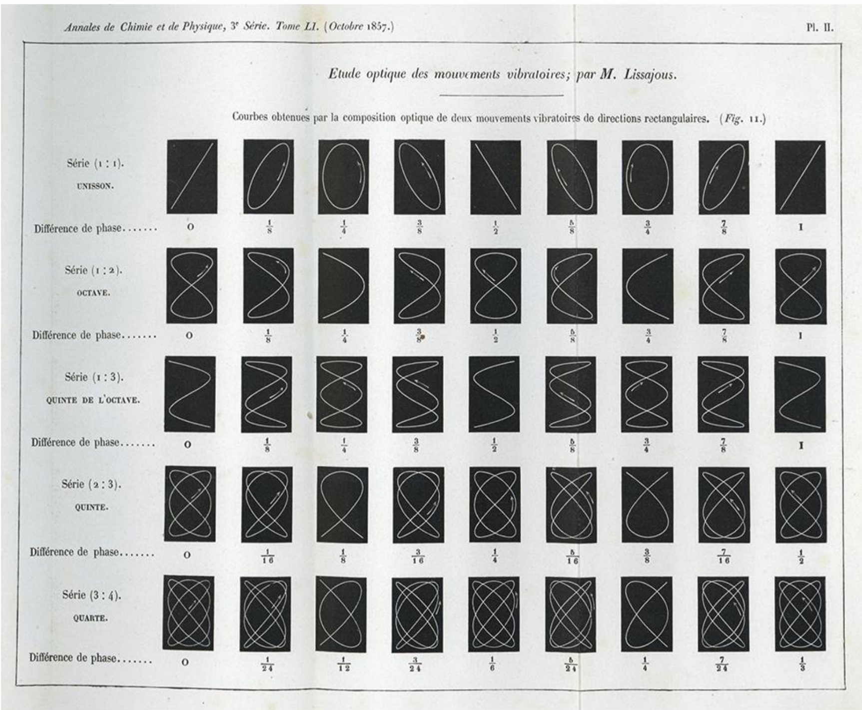
The Lissajous figures uses mainly consists of measurement of the frequency and measurement of the phase difference. The Lissajous figure is of high importance in physics in order to study the sinusoidal waves. The Lissajous figures are mainly used in analogue electronics to analyse the intersection of two or more sinusoidal wave constructing loops which are also known as Lissajous knots in general. They are the shapes created when the x-coordinate of a curve is described by one sine wave, and the y-coordinate is described by another sine wave. By adjusting the frequency of each wave, the phase between them, and their relative amplitudes, interesting patterns emerge.

# CHAPTER 2
# LITERATURE SURVEY

Jules Antoine Lissajous (1822–1880) became known to the scientific society with his analysis on “Lissajous figures”—patterns formed when two vibrations along perpendicular lines are superimposed. Lissajous entered the ´Ecole Normale Sup´erieure in 1841 and later became professor of physics at the Lyc´ee Saint-Louis in Paris, where he studied vibrations and sound.

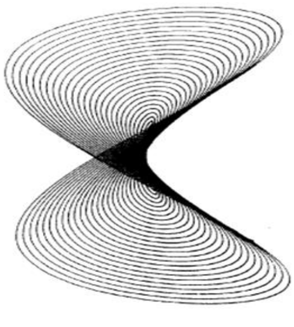
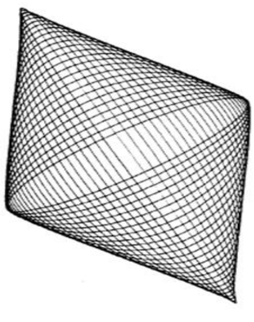
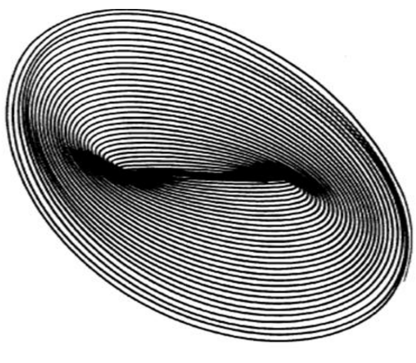
Like some other physicists of his time, Lissajous was interested in demonstrations of vibration that did not depend on the sense of hearing. His most important research, first described in 1855, was the invention of a way to study acoustic vibrations by reflecting a light beam from the vibrating object onto a screen. Lissajous produced two kinds of luminous curves. In the first kind, light is reflected from a tuning fork (to which a small mirror is attached), and then from a large mirror that is rotated rapidly. The second kind of curve, named the 'Lissajous figure,' is more useful. The light beam is successively reflected from mirrors on two forks that are vibrating about mutually perpendicular axes. Persistence of vision causes various curves, whose shapes depend on the relative frequency, phase, and amplitude of the forks' vibrations. If one of the forks is a standard, the form of the curve enables an estimate of the parameters of the other. As Lissajous said, they enable one to study beats (the ellipses rotate as the phase difference changes). 'Lissajous figures' have been, and still are, important in this respect

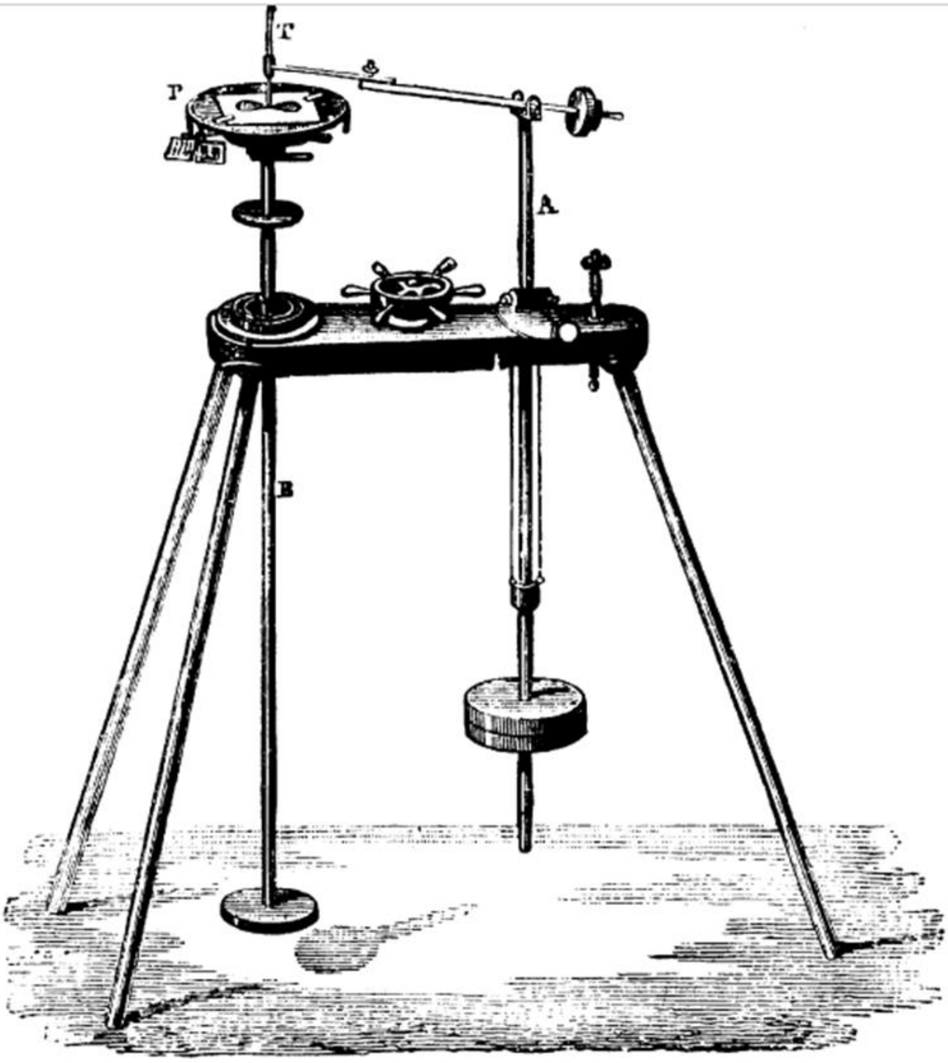
In 1857, he published "Mémoire sur l'Etude optique des mouvements vibratoires," *Annales de chimie et de physique*, 3rd series, 51 (1857) 147-232, 2 folding plates. Lissajous's paper on his optical method of studying vibration gave rise to the widely used "Lissajous figures," or Lissajous curves defined mathematically as curves in the xy plane generated by the functions $y = a \sin (w_1t + q_1)$ and $x = b \sin (w_2t + q_2)$ where $w_1$ and $w_2$ are small integers.




Lissajous figures from Lissajous's original publication on the subject.

Lissajous’ work was praised by his contemporaries and discussed by the physicists John Tyndall (1820–1893) and Lord Rayleigh (John William Strutt, 1842–1919) in their classical treatises on acoustics. In 1873 he was awarded the prestigious Lacaze Prize for his “beautiful experiments,” and his method was exhibited at the Paris Universal Exposition in 1867. But apparently nothing is new under the sun: Lissajous’ figures had been discovered long before by the self-taught American scientist Nathaniel Bowditch (1773–1838). Bowditch also wrote many scientific papers, one of which, on the motion of a pendulum swinging simultaneously about two axes at right angles (to illustrate the apparent motion of the Earth as viewed from the Moon), described the so-called Bowditch curves (better known as the Lissajous figures, after the man who later studied them in detail). He produced these patterns in 1815 with a compound pendulum.



A variation of this device, in which the motion of two pendulums is combined and traced on paper by means of a pen attached to one of the pendulums, became a popular nineteenth-century science demonstration.

The ensuing figures were called “harmonograms,” and their incredible variety never failed to impress the spectators. The novelty of Lissajous’ method was that it departed from mechanical devices and relied instead on the much more efficient agent of light. In this he was a visionary, foretelling our modern electronic era.

# CHAPTER 3

# LISSAJOUS PATTERN

A Lissajous figure is produced by taking two sine waves and displaying them at right angles to each other. This easily done on an oscilloscope in XY mode. If the oscilloscope has the x-versus-y capability, one can apply one signal to the vertical deflection plates while applying a second signal to the horizontal deflection plates. The horizontal sweep section is automatically disengaged at this time. The resulting waveform is called Lissajous figure. This mode can be used to measure phase or frequency relationships between two signals.

When the two sine waves are of equal frequency and in-phase, diagonal line to the right will produced. When the two sine waves are of equal frequency and 180 degrees out-of-phase a diagonal line to the left will produced. When the two sine waves are of equal frequency and 90 degrees out-of-phase a circle will produced. When the horizontal and vertical sine wave frequencies differ by a fixed amount, this is equivalent to constantly rotating the phase between them.

## 3.1 Phase Measurements Using Lissajous Figures

Lissajous figures are sometimes used for the measurement of phase. Lissajous figures are produced in an oscilloscope by connecting one signal to the vertical trace and the other to the horizontal trace. If the ratio of the first frequency to the second is a rational number, then a closed curve will be observed on the CRO. If the two frequencies are unrelated, then there will be only a patch of light observed because of the persistence of the oscilloscope screen.

If the two signals have the same frequency, then the Lissajous figure will assume the shape of an ellipse. The ellipse’s shape will vary according to the phase difference between the two signals, and according to the ratio of the amplitude of the two signals.

For few Lissajous figures based on their shape, we can directly tell the phase difference between the two sinusoidal signals.

- If the Lissajous figure is a straight line with an inclination of 45° with positive x-axis, then the phase difference between the two sinusoidal signals will be 0°. That means, there is no phase difference between those two sinusoidal signals.
- If the Lissajous figure is a straight line with an inclination of 135° with positive x-axis, then the phase difference between the two sinusoidal signals will be 180°. That means, those two sinusoidal signals are out of phase.

- If the Lissajous figure is in circular shape, then the phase difference between the two sinusoidal signals will be 90° or 270°.

We can calculate the phase difference between the two sinusoidal signals by using formulae, when the Lissajous figures are of elliptical shape.

- If the major axis of an elliptical shape Lissajous figure having an inclination angle lies between 0° and 90° with positive x-axis, then the phase difference between the two sinusoidal signals will be.

$$\phi = \sin^{-1}\frac{x_1}{x_2} = \sin^{-1}\frac{y_1}{y_2}$$

If the major axis of an elliptical shape Lissajous figure having an inclination angle lies between 90° and 180° with positive x-axis, then the phase difference between the two sinusoidal signals will be

$$\phi = 180 - \sin^{-1}\frac{x_1}{x_2} = 180 - \sin^{-1}\frac{y_1}{y_2}$$

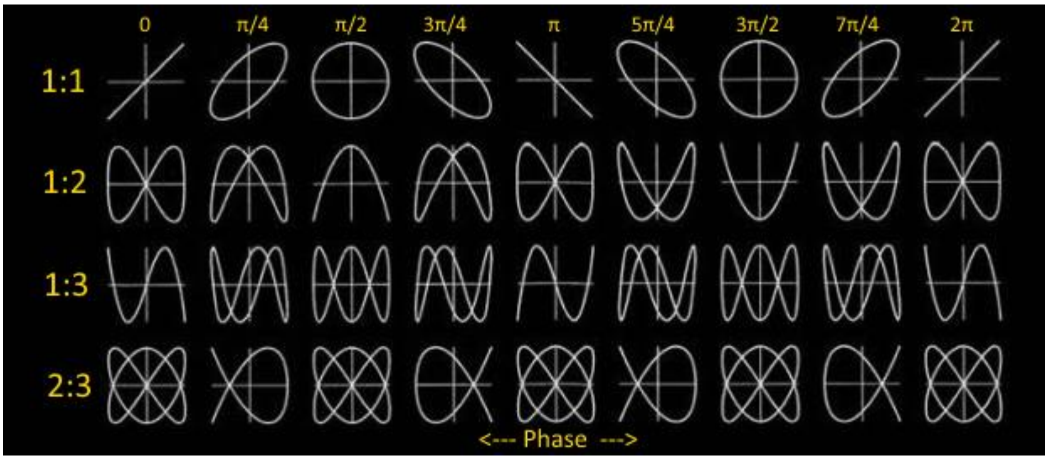
Where,

$x_1$ is the distance from the origin to the point on x-axis, where the elliptical shape Lissajous figure intersects.

$x_2$ is the distance from the origin to the vertical tangent of elliptical shape Lissajous figure.

$y_1$ is the distance from the origin to the point on y-axis, where the elliptical shape Lissajous figure intersects.

$y_2$ is the distance from the origin to the horizontal tangent of elliptical shape Lissajous figure



## 3.2 Frequency Measurements Using Lissajous Figures

Lissajous figure will be displayed on the screen, when the sinusoidal signals are applied to both horizontal & vertical deflection plates of CRO. We can apply the sinusoidal signal, which has standard known frequency to the horizontal deflection plates of CRO. Similarly, apply the sinusoidal signal, whose frequency is unknown to the vertical deflection plates of CRO

Let, $f_H$ and $f_V$ be the frequencies of sinusoidal signals, which are applied to the horizontal & vertical deflection plates of CRO respectively. The relationship between $f_H$ and $f_V$ can be mathematically represented as below.

$$f_V/f_H = n_H/n_V$$

From above relation, we will get the frequency of sinusoidal signal, which is applied to the vertical deflection plates of CRO as

$$f_V = (n_H/n_V)\, f_H \qquad \text{-(1)}$$

Where,

$n_H$ is the number of horizontal tangencies and

$n_V$ is the number of vertical tangencies.

Two lines are drawn, one vertical and one horizontal so that they do not pass through any intersection on Lissajous pattern. Then the number of intersections of the horizontal and vertical lines with the Lissajous patterns and counted separately. So after finding the tangencies if we know we can easily calculate the unknown frequency applied to vertical plate.

By substituting the values of $n_H$, $n_V$ and $f_H$in (1), we will get the value of $f_V$, i.e. the frequency of sinusoidal signal that is applied to the vertical deflection plates of CRO.

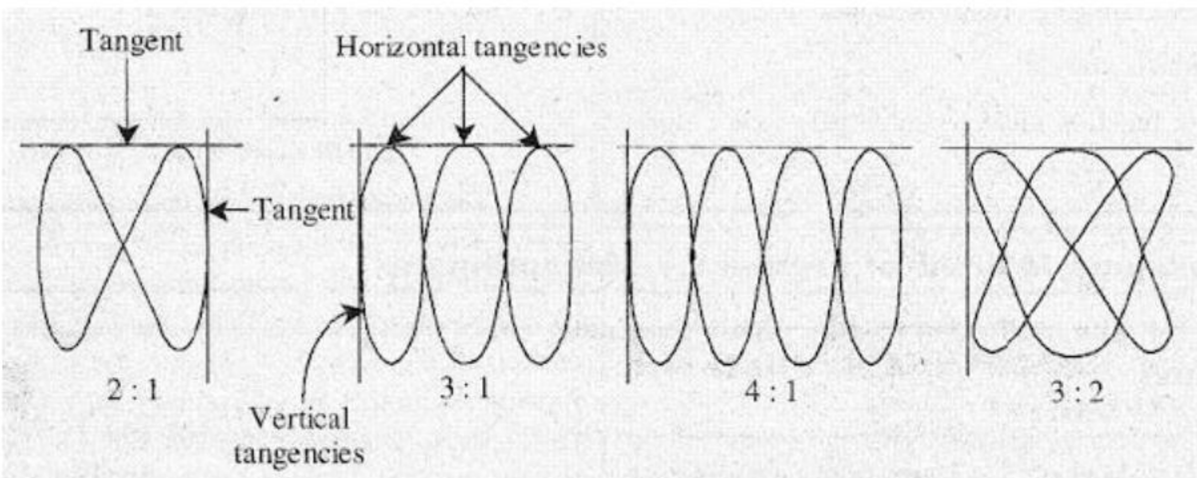


fig 6.2 Lissajous patterns allowing different frequencies ratios

All electronic circuits in the oscilloscope like attenuators, time base generators, amplifiers cause some amount of time delay while transmitting signal voltage to deflection plates. We also know that horizontal signal is initiated or triggered by some portion of output signal applied to vertical plates of CRT. So the delay line is used to delay the signal for some time in the vertical section of CRT.

## 3.3 Lissajous Figures Using Compound Pendulum

A device consisting of two coupled pendula, usually oscillating at right angles to each other, which are attached to a pen. The resulting motion can produce beautiful, complicated curves which eventually terminate in a point as the motion of the pendula is damped by friction. In the absence of friction (and for small displacements so that the general pendulum equations of motion become simple harmonic motion), the figures produced by a harmonograph would be Lissajous curves.

Lissajous curves are a special case of the harmonograph with damping constants $\beta_1=\beta_2=0$.

The compound pendulum may be made to swing in two different planes and to swing with two different periods. One plane of swing is fixed and is determined by the supporting of the two cords. The other plane of swing may be made anything desired by the operator but is usually made to swing at right angles to the upper plane. The period of the swing of the lower pendulum and the upper pendulum depends upon the location of the movable collar. The periods of the two swings may thus be adjusted to any desired ratio. some very interesting patterns may be obtained which illustrate the result of combining two harmonic motions of different periods.

# CHAPTER 4

# EXPERIMENT

## 4.1 LISSAJOUS PATTERNS USING PENDULUM METHOD

We attempt to create Lissajous patterns using a compound pendulum and an LED light source. Different patterns can be obtained by varying the length of the 2 parts of the pendulum.

**APPARATUS:**

LED light source

Transparent wire (fishing line)

A small weight

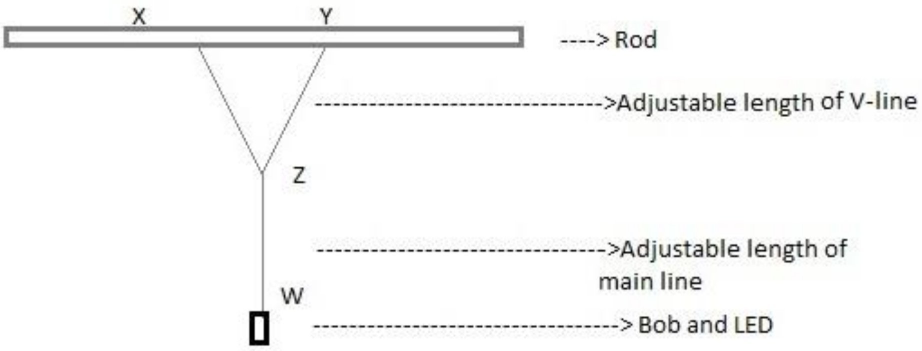
A long rod to hang the pendulum

Camera with adjustable shutter speed

Tripod stand

**PROCEDURE**

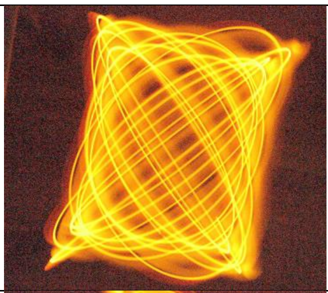
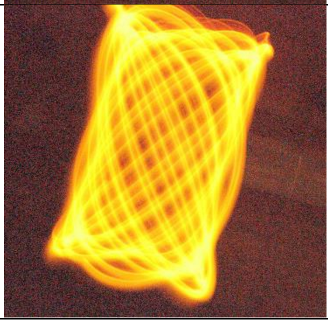
The long rod is set up firmly and from two measured points, the wire or line is attached and brought to meet in the middle in the form of a V. From the tip of the V, the main length of the pendulum is attached. At the tip of this line, the LED, attached to a suitable weight (which will not be changed for the duration of the experiment) is strung on. A tripod stand is used to position the camera above the pendulum in such a way that it can capture the movement of the pendulum without anything else coming into its field of view. The shutter speed of the camera is set to 30 seconds and focussed as required. The length of the upper region of the pendulum as well as that of the line attached to the bob is measured in advance. The pendulum is set into circular motion and the camera is used to capture the long exposure shot and the pattern is recorded. The process is repeated by varying the lengths of the upper portion of the pendulum as well as that of the main line. The patterns are compared to the ones obtained using an oscilloscope.

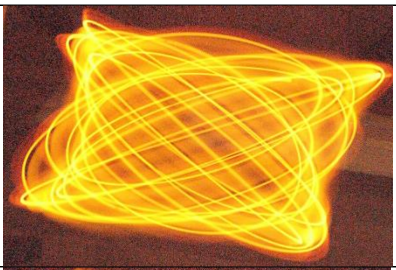
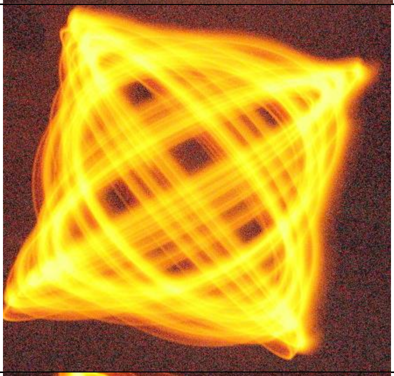
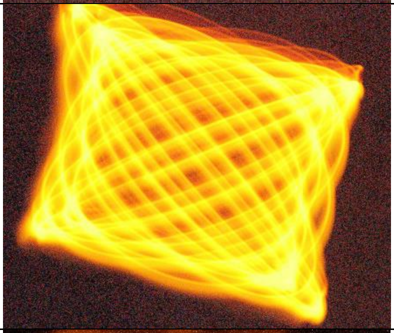
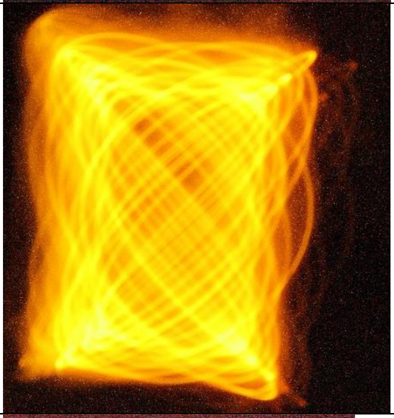
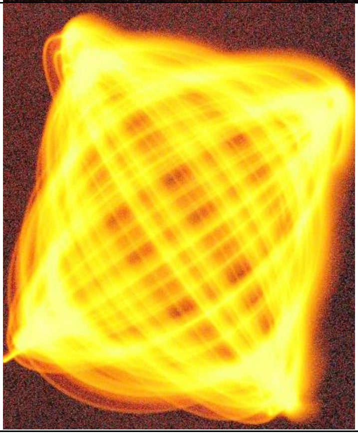


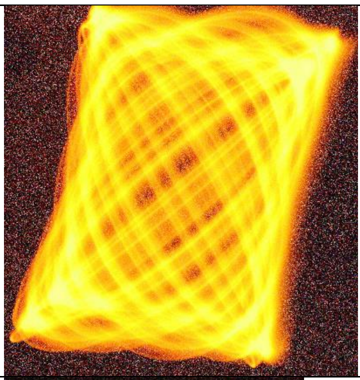
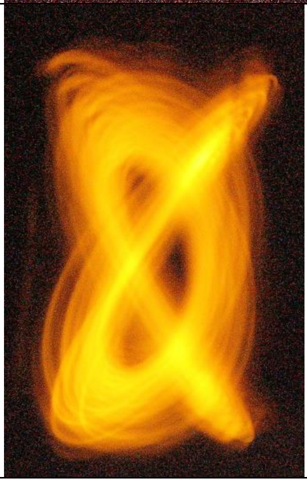

**PRECAUTIONS**

- The wire used must be transparent so that it doesn't obstruct the camera.
- The rod and tripod must be kept steady.
- The room in which the experiment is done must be kept dark to get a more accurate patterns.
- Take care to set the pendulum in almost the same motion (without excessive force and from same position as much as possible).

**OBSERVATIONS**

| **Length of V Pendulum A (cm)** | **Length of Main line B (cm)** | **Ratio A:B** | **Obtained pattern** |
|---|---|---|---|
| 50 | 100 | 1:2 |  |
| 50 | 80 | 5:8 |  |

| | | | |
|---|---|---|---|
| 30 | 100 | 3:10 |  |
| 30 | 80 | 3:8 |  |
| 30 | 60 | 1:2 |  |
| 30 | 20 | 3:2 |  |
| 20 | 100 | 1:5 |  |

| | | | |
|---|---|---|---|
| 20 | 80 | 1:4 |  |
| 20 | 20 | 1:1 |  |

**4.2 FREQUENCY MEASUREMENT USING LISSAJOUS FIGURES**

We attempt to create Lissajous Patterns using oscilloscope and verify the theory relating the frequency ratio to the ratio of number of vertical and horizontal tangents.

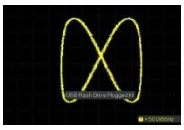
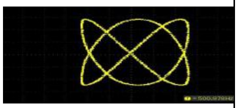
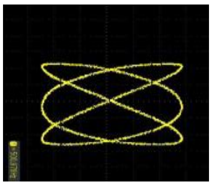
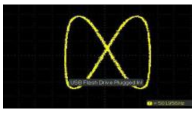
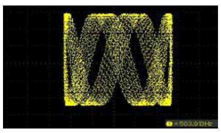
**APPARATUS**

1. Two signal generators
2. Oscilloscope

**PROCEDURE**

Two signal generators are used and they are connected to the vertical and horizontal traces of the oscilloscope. The signal voltage is set at 4V peak-to-peak. Vary the frequency of the generators until a closed pattern occurs on the scope. This pattern is recorded. The number of vertical and horizontal tangencies and their ratio as well as frequency ratio of the generators is calculated (vertical tangencies are points where the figure is tangent to the vertical axis. Similar for horizontal). Procedure is repeated for other frequency values and experiment is used to verify the theory.

## OBSERVATIONS

| Lissajous Figure | No. Of Vertical Tangencies $n_H$ | No. Of Horizontal Tangencies $n_V$ | $n_H/n_V$ | Frequency of Generator 1 (Hz) $f_V$ | Frequency of Generator 2 (Hz) $f_H$ | $f_V/f_H$ |
|---|---|---|---|---|---|---|
|  | 1 | 2 | 1:2 | 1 | 2 | 1:2 |
|  | 2 | 3 | 2:3 | 1 | 1.5 | 2:3 |
|  | 3 | 2 | 3:2 | 3 | 2 | 3:2 |
|  | 1 | 2 | 1:2 | 0.5 | 1 | 1:2 |
|  | 1 | 4 | 1:4 | 0.5 | 2 | 1:4 |

# CHAPTER 5
# CONCLUSIONS

Lissajous patterns are the shapes created when the x-coordinate of a curve is described by one sine wave, and the y-coordinate is described by another sine wave. By adjusting the frequency of each wave, the phase between them, and their relative amplitudes, interesting patterns emerge. They can be created using oscilloscopes and compound pendulums.

Lissajous pattern can be obtained using a swinging pendulum. The use of 2 strands of string gives the pendulum a more dynamic motion owing to the altered time periods as it moves on two perpendicular axes at the same time. Since the light lines are not distinct enough and is harder to align along a line, accurate frequency calculation using this method is not recommendable.

From the second experiment, we verified that the ratio of number of vertical tangencies to number of horizontal tangencies are equal to ratio of the two input frequencies. So we can conclude that Lissajous Figures are affected by the ratio of the two input frequencies. This implies that Lissajous figures provide a way to find an unknown frequency. Additionally, it was seen that for high frequency input, the given pattern moves faster. And so, Lissajous Figure also depends on value of frequency input.

# CHAPTER 6
# APPLICATION OF LISSAJOUS FIGURES

Lissajous figures are used for real-time analysis of the phase relationship between the left and right channels of a stereo audio signal. A Lissajous curve is used in experimental tests to determine if a device may be properly categorized as a memristor. It is also used to compare two different electrical signals: a known reference signal and a signal to be tested.

Creation of simple Lissajous figures during rewarming for bypass runs may be an additional helpful tool in root cause analysis of patient death/morbidity due to unexplained organ failure post major aortic surgery, when surgery, perfusion, and anesthesia seem faultless. A simple graphical representation of the large amounts of electronic perfusion data on mixed venous saturation and temperature that is generated during prolonged cases may aid analysis. "Unknown" reasons for death/morbidity usually relate to organ ischemia and inflammation. The introduction of electronic data management (EDM) systems enables accurate and detailed collection of perfusion information. Prior to this, recorded information was infrequent and subjected to error and bias by clinicians. Electronic acquisition systems make it possible to capture hundreds of variables as frequently as every 20 seconds with great accuracy. Nasopharyngeal temperature and mixed venous oxygen saturations are plotted to potentially identify organ ischemia which may help to explain why some deaths/morbidities occur.

Lissajous figures were sometimes displayed on oscilloscopes meant to simulate high-tech equipment in science-fiction TV shows and movies in the 1960s and 1970s. They are sometimes used in graphic design as logos.

Examples include:

- The Lincoln Laboratory at MIT ($a = 3$, $b = 4$, $\delta = \pi/2$)
- Disney's Movies Anywhere streaming video application uses a stylized version of the curve

- Facebook's rebrand into Meta Platforms is also a Lissajous Curve, echoing the shape of a capital letter M ($a = 1$, $b = -2$, $\delta = \pi/20$)

The Dadaist artist Max Ernst painted Lissajous figures directly by swinging a punctured bucket of paint over a canvas. Interesting patterns can be made using light, paint and sand which reveal the elegance and artistry of these mathematical equations.

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