COUPLED PENDULUM

PROJECT REPORT

Submitted by

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ST.TERESA'S COLLEGE (AUTONOMOUS)

ERNAKULAM

CERTIFICATE

This is to certify that the project report entitled **"COUPLED PENDULUM "** is a bonafide work by Tania Mary Ninan(Reg no. AB19PHY036) of St.Teresa's College Ernakulam, under my supervision at the Department of Physics, St.Teresa's College, Ernakulam for the partial fulfilment of the award of Degree Of Bachelor of Science in Physics during the academic year 2020-'21 .The work presented in this dissertation has not been submitted for any other degree in this or any other university.

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Place:Ernakulam

Date: 9/05/2022.

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DECLARATION

I,Tania Mary Ninan(Register Number:AB19PHY036), final year B.Sc. Physics student, Department of Physics, St.Teresa's College, Ernakulam do hereby declare that the project work entitled **"COUPLED PENDULUM"** has been originally carried out under the guidance and supervision of Dr. Mariyam Thomas, Assistant Professor, Department of Physics, St.Teresa's College (Autonomous), Ernakulam in partial fulfilment for the award of the Degree of Bachelor of Physics. I further declare that this project is not partially or wholly submitted for any other purpose and the data included in the project is collected from various sources and is true to the best of my knowledge.

Place: Ernakulam

Date: 9/5/2022.

Tania Mary Ninan

ACKNOWLEDGMENT

I am bringing out this project report with immense pleasure and sense of satisfaction. I feel obliged to acknowledge the support and guidance that came from various quarters during the course of competition of this project. I would like to express our sincere gratitude to Dr. Mariyam Thomas, the project in charge , for guiding me throughout the entire duration of the project work. This work would not have succeeded without her support and motivation.

I would also like to extend our sincere thanks to all the faculty members of the Physics Department for their valuable suggestions and corrections. I would also like to thank all our friends for being with me whenever I was in need. Above all, I owe my heartfelt gratitude to the Almighty for showers abundant blessings upon me to get through this project.

ABSTRACT

The main objective of this project is to analyse the working of a coupled pendulum with two identical point masses coupled using a string and a weight suspended in the middle.The time period of oscillations of different masses and that of different phase differences between the two pendulums are recorded. Inferences are made from above observations. Also to study and analyse Lissajous pattern obtained using a coupled pendulum.

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CHAPTER 1
INTRODUCTION

A physical pendulum is defined as a rigid body that can perform rotary oscillation around a fixed axis under influence of the gravitational force.Two pendulums that can exchange energy are called coupled pendulums.There is a definite fascination to coupled oscillations, but it is hard to find a mechanical system which is both easy to build and easy to analyse. A system of one-dimensional alternating springs and masses, for example, (three springs and two masses) is easy to analyse, but not easy to build. The coupled-oscillation system used in this experiment, however, is trivial to construct, and not too hard to analyse. It is made from light string or thread, and two equal masses. The intended oscillations of this system are in and out of the plane of the paper.In this experiment, both oscillations in phase and opposite in phase are studied.

This phenomenon can be used to show balanced and unbalanced forces, how motion can be used to predict future motion, and the conservation of energy. If a physicist or physical chemist wants to calculate the wavelengths of infrared (thermal) radiation that a gaseous substance is able to absorb and emit, they will use the exact same mathematics that are used when calculating the normal modes of a coupled pendulum system. This is because molecular thermal vibrations can be seen as a type of coupled oscillator system. Hence it finds numerous applications.

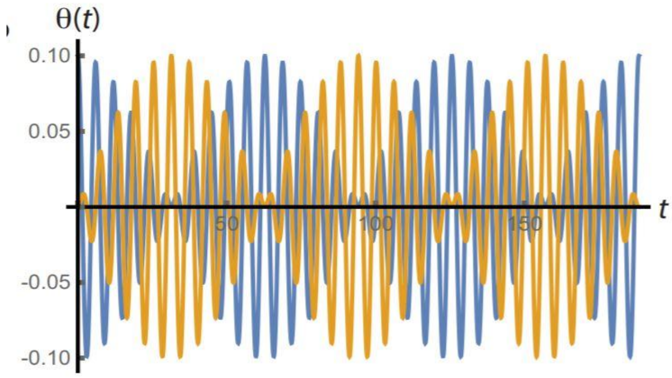
1.1 THEORY

MOTION OF THE PENDULUM

The coupled pendulum system under analysis is shown in figure. It consists of two cylindrical bobs suspended from a fixed support by two thin strings of the same length l. Each bob can oscillate independently and represents a simple pendulum. These two independent pendulums can be coupled by a thin rod or a string from which a mass is suspended, around which each string is wrapped. The coupling string is positioned horizontally at height h with respect to the centre of mass of the two bobs. Such a system can be made to oscillate by displacing one of the bobs from its equilibrium position, in a vertical plane containing the mass, and then releasing it. The motion of this bob transfers to the other one. Over time, the amplitude of the first bob's oscillation decreases to a minimum amplitude as the other bob oscillates with increasing amplitude. The process is then reversed: the amplitude of the second bob decreases until it reaches a minimum amplitude, as the first bob increases its amplitude to a maximum. This interchange of motions continues until the two pendulums come practically to rest, because of unavoidable dissipative forces. The motion of the two bobs is coupled: energy transfer from one pendulum to the other is clearly due to the coupling rod. Moreover, the time interval between two consecutive minimum amplitudes of a given bob, or in other words, the time needed to transfer the energy of the motion, depends on the string's position, i.e. on the height h: the time increases as h approaches the pendulums' length l.

The energy transfer is a beat frequency – caused by the interaction between the two swings. Every time pendulum A swings, it pushes on the string, which in turn pushes on Pendulum B –transmitting energy to pendulum B. According to Newton's 3rd law (equal and opposite force), pendulum B pushes back on Pendulum A, slowing it down and taking energy from it! This happens very regularly –so regularly it can be calculated – since it is essentially two pendulums – one the full length, and one from where the individual pendulums contact the string

Since the two pendulums have the same length, the pulls of the first pendulum on the second occur exactly at the natural frequency of the second pendulum, so it (the second pendulum) begins to swing too. However, the second pendulum will swing slightly out of phase with the first one. When the first pendulum is at the height of its swing, the second pendulum is still somewhere in the middle of its swing.

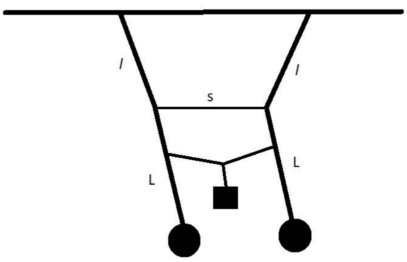


In the given experimental set we have three oscillating pendulums in effect, i.e., two pendulums at the ends having the same length and the mass attached to the string. During the energy transfer in the Coupled pendulum, one of the pendulums at the end begins to oscillate keeping others stationary. As time goes on the pendulum begins to slow down and the mass hanging on the string gets into oscillation. This energy is transferred to the third pendulum . As the third pendulum gains the maximum amplitude the first becomes stationary for some time. This energy transfer continues till the dissipative forces stops the oscillating pendulum .

MODES OF VIBRATION

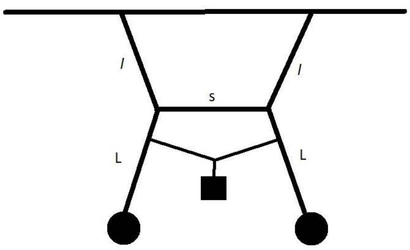
First mode: If we draw the two masses aside some distance and release them simultaneously from rest, they will swing in identical phase with no relative change in position. The spring will remain unstretched (or uncompressed) and will exert no force on either masses.

This is called the first mode of vibration of the system.



Second mode: The other obvious way of starting a symmetric oscillation will be to stretch the spring from both ends. If we release the masses from rest simultaneously, we may notice that: a) The spring now exerts forces during motion b) From symmetry of motions of A and B, their positions are mirror images of each .

This is called the second mode of vibration of the system.



1.2 IMPORTANCE OF COUPLED PENDULUM

The double pendulum system exhibits rich dynamical behaviour. This makes it a perfect candidate for the study of chaotic motion, as it provides the quintessential example of a simple physical system that can display surprisingly complex motion under certain conditions.

Pendulums are used in many engineered objects, such as clocks, metronomes, amusement park rides and earthquake seismometers. In addition, engineers know that understanding the physics of how pendulums behave is an important step towards understanding motion, gravity, inertia and centripetal force.

CHAPTER 2

MATERIALS AND METHODOLOGY

2.1 MATERIALS

Part A: Lissajous Pattern obtained by sand pendulum

To construct a Sand pendulum we attach a string to two rigid supports A and B . A plastic bottle is suspended from the string connected to A and B. The bottle is tied to a string as shown in figure (b). A small hole is made at the bottom of the bottle so that the sand can flow continuously through it

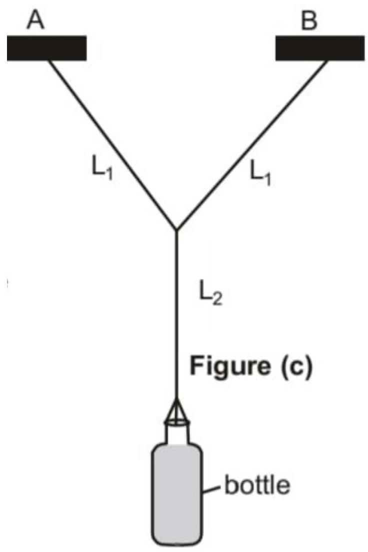


Figure (c)

Part B : Study on damping effects the coupled pendulum

The coupled pendulum we used for this project was constructed using simple household items available easily.

- Two rubber balls were taken as the bob of the pendulum- considered as point masses

- Strings to connect the bobs
- Heavy weights and hook in the centre

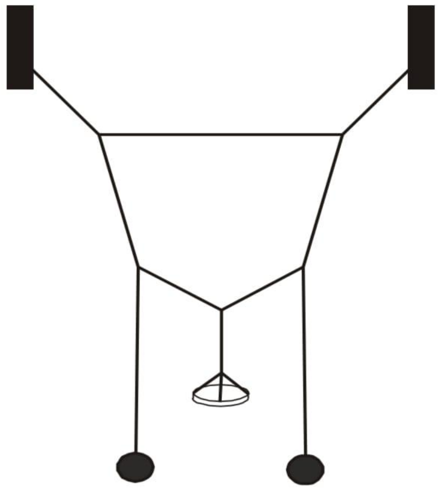


Fig.1-Experimental set up

2.2 METHODOLOGY

If two simple pendulums are connected to each other by a string/wire, the oscillations of one pendulum will affect the other. Such an arrangement is called a coupled pendulum.

- **Lissajous Pattern obtained by sand pendulum**

For doing this part of the experiment, you have to create a new setup which is described below

The experimental Setup: The string to two rigid supports A and B as shown in figure below. A plastic bottle is to be suspended from the string connected to A and B. Make appropriate arrangements to tie the bottle to a string as shown in figure (b). Make sure the bottle is nearly vertical. Make a small hole at the bottom of the bottle. Now fill the bottle with sand/salt or any other free flowing material. Check that the hole is of appropriate size so that the material filled in it can continuously come out. Close the hole using a cello tape, fill it with your material and suspend it as shown in figure (e). The bottle may be at an appropriate height from a horizontal platform where the sand can fall and make a pattern.

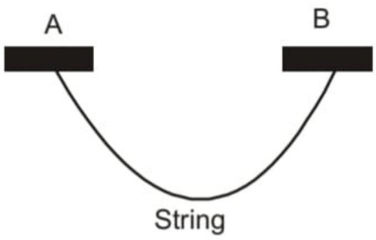


Figure (a)

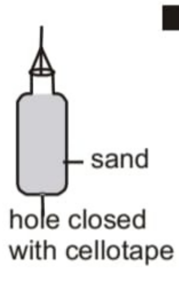


Figure (b)

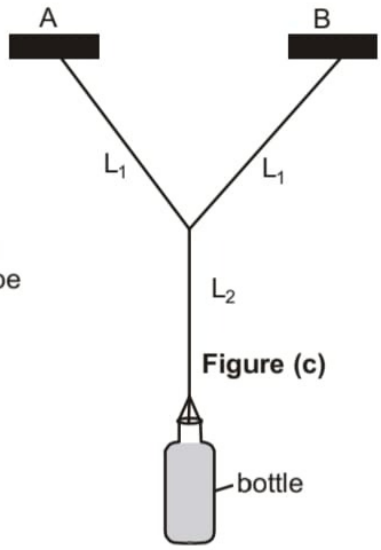


Figure (c)

- **Study on damping effects the coupled pendulum**

Making the coupled pendulum:

To make the coupled pendulum, you will require strings and two identical solid objects. Fasten the string to two vertical stands such that the string is horizontal. Now make two identical simple pendulums and tie them to the string at a certain separation.

Tie another short string to the two individual pendulums and suspend a pan balance/weight from the middle of this tying t string. The whole setup is schematically shown in the figure.

- Make your own pan balance

You may use small plastic plates/cups and tie them with strings to create the balance. Think of ways to make homemade weights which can be varied in equal steps.

★ Measure time period **T1:**

Keeping the initial phase difference between the pendulums to be zero, oscillate them and find the time period of these oscillations.

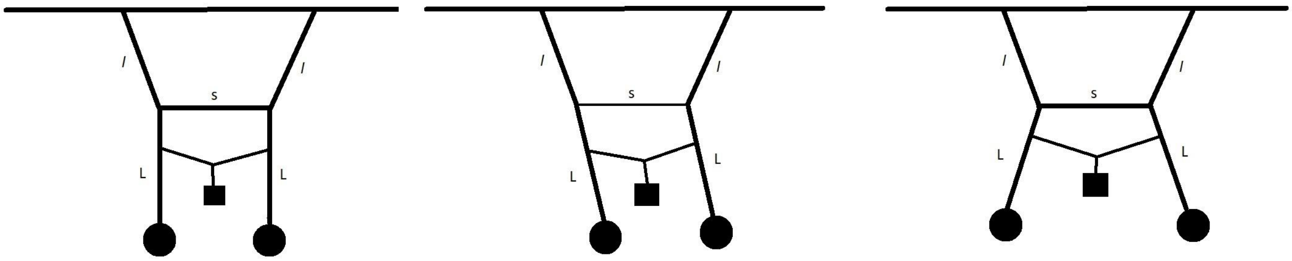
★ Measure time period **T2:**

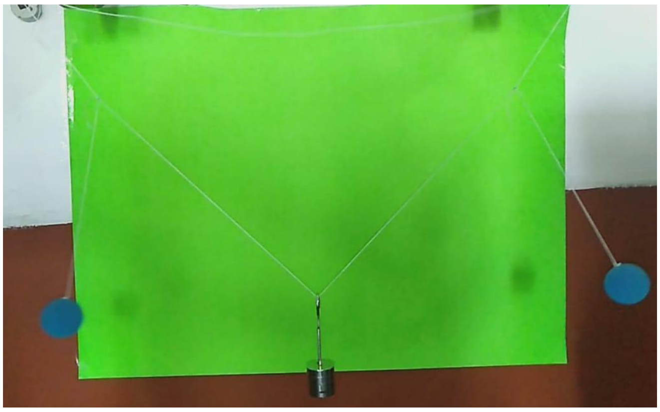
Keeping the initial phase difference between the pendulum to be 180 degrees oscillates the pendulums and find the time period of these oscillations.

★ Measure time period **T3:**

Keeping the first pendulum stationary,displace the second pendulum through a distance and release. You will notice that its oscillations die down after some time and the first pendulum picks up oscillations gradually. After some time the oscillations of the first pendulum die down and the second pendulum gradually starts oscillating, and then it stops again. This process continues for some time.The time between the consecutive stops of the second (or first) pendulum is T3.

Find T_1, T2. and T3 for various weights in the range of about 0-100g. Tabulate and analyse the data obtained. Plot the variation of T1 T2 and T3 with weights in the pan.





Oscillations

Let the vertical plane through the strings be called plane M. Pull the bottle on this planet through a certain distance and release Study the oscillations of the bottle.

Next, pull the bottle perpendicular to plane M through a certain distance and release. Study the oscillations. Now pull the bottle at an angle of 45 degrees with plane M and release. Again study the oscillations

Study the sand patterns

Pull the bottle at an angle of 30 degrees from the plane M. Remove the tape from the bottom of the bottle and release. Describe the pattern formed by the falling sand. Repeat it by oscillating the bottle at angles of 45 and 60 degrees with the plane M. Record the patterns formed.

CHAPTER 3

RESULTS AND DISCUSSIONS

3.1 OBSERVATIONS

- *0 Phase difference between the pendulums*

Mass in the centre	Time for 10 oscillations of pendulum 1 (s)	Time for 10 oscillations of pendulum 2 (s)	Time period of pendulum1 T1 (s)	Time period of pendulum 2 T2 (s)
50g	11.76	11.98		
	11.84	11.95	1.1823	1.186
	11.87	11.64		
100g	11.37	11.11		
	11.31	11.16	1.131	1.114
	11.25	11.16		
150g	10.73	10.90		
	10.90	10.92	1.085	1.090
	10.92	10.90		
200g	10.86	10.63		
	10.82	10.65	1.081	1.061
	10.75	10.57		

- *180 degree phase difference between the pendulums*

Mass in the centre	Time for 10 oscillations of pendulum 1 T1(s)	Time for 10 oscillations of pendulum 2 T2(s)	Time period of pendulum1 T1 (s)	Time period of pendulum 2 T2 (s)
50g	11.14	11.21		
	11.33	11.43	1.112	1.122
	10.89	11.03		
100g	10.74	10.87		
	10.89	10.82	1.08	1.083
	10.77	10.80		
150g	10.66	10.58		
	10.60	10.73	1.065	1.062
	10.70	10.56		
200g	10.43	10.50		
	10.46	10.49	1.046	1.054
	10.51	10.63		

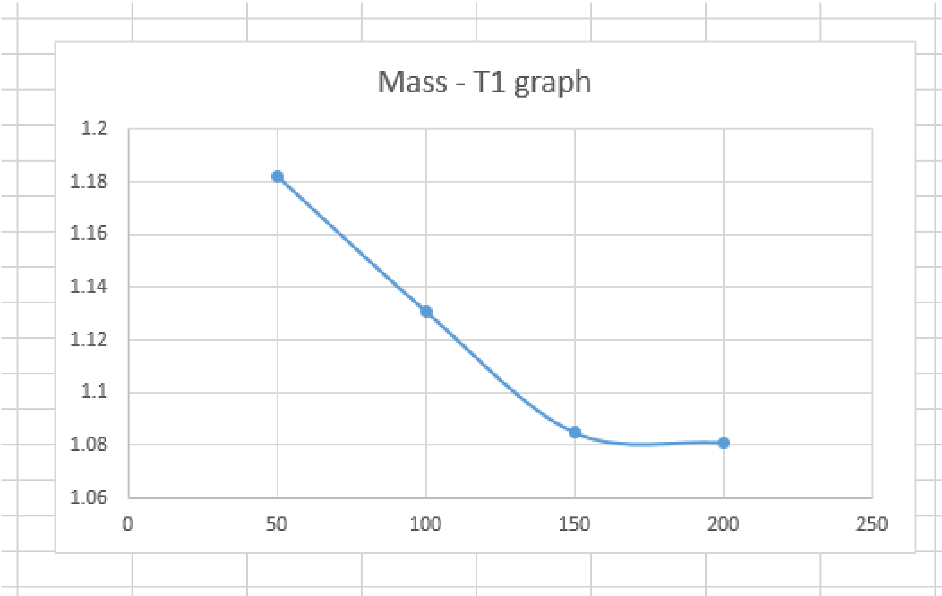
- *When only one bob is oscillating*

Mass in the centre	Time for 3 oscillations of pendulum 1 T1(s)	Time for 3 oscillations of pendulum 2 T2(s)	Time period of pendulum1 T1 (s)	Time period of pendulum 2 T2 (s)
50g	3.52	3.49		
	3.57	3.6	1.19	1.17
	3.62	3.47		
100g	3.16	3.25		
	3.27	3.33	1.074	1.096
	3.24	3.29		
150g	3.17	3.16		
	3.16	3.19	1.052	1.057
	3.14	3.17		
200g	3.08	3.09		
	3.07	3.06	1.017	1.024
	3.01	3.07		

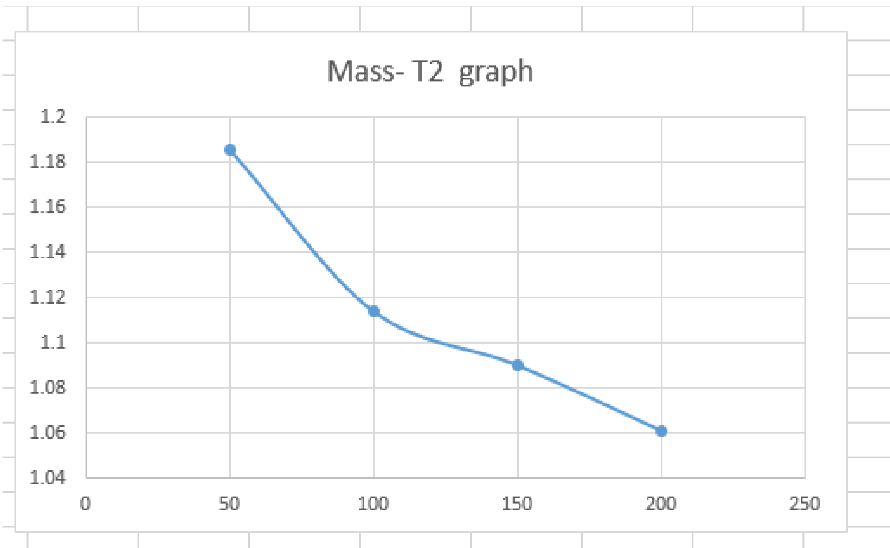
3.2 GRAPHS

- **<u>0 Phase difference between the pendulums</u>**

Graph plotted between mass in the centre and time period of pendulum 1

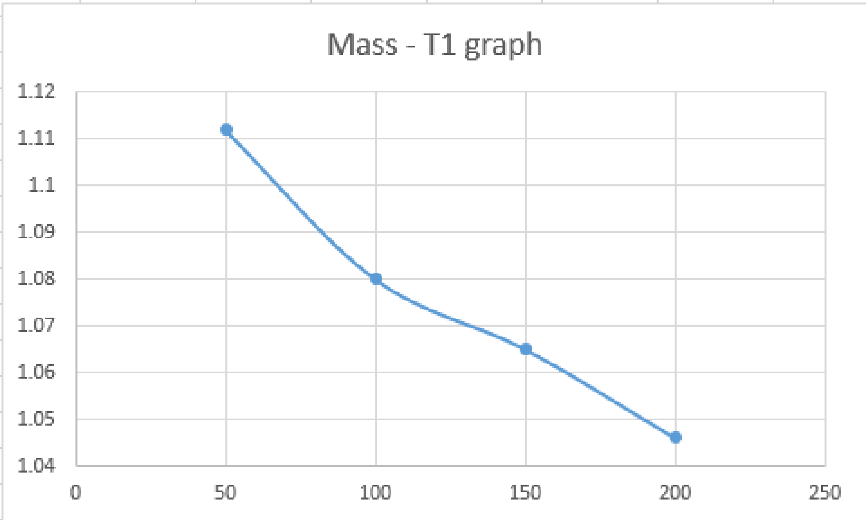


Graph plotted between mass in the centre and time period of pendulum 2

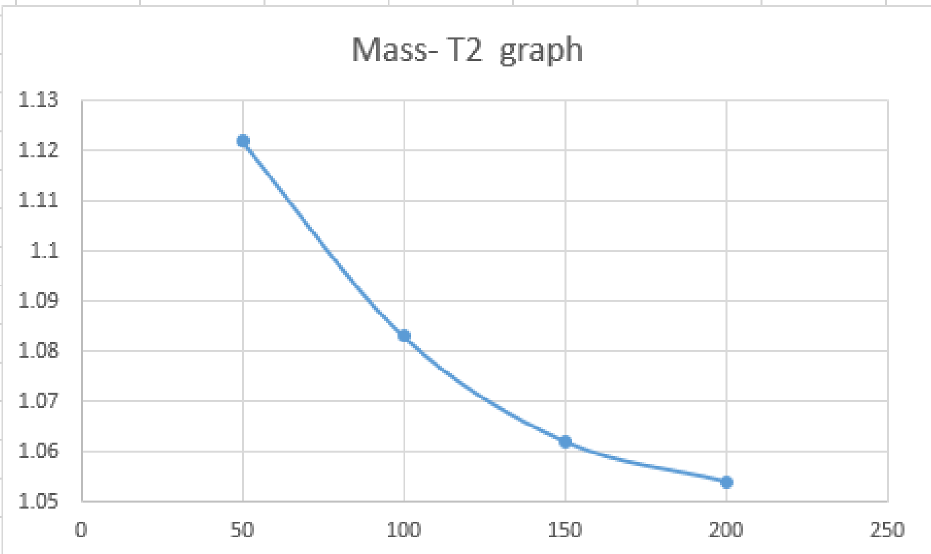


- **<u>180 degree phase difference between the pendulums</u>**

Graph plotted between mass in the centre and time period of pendulum 1

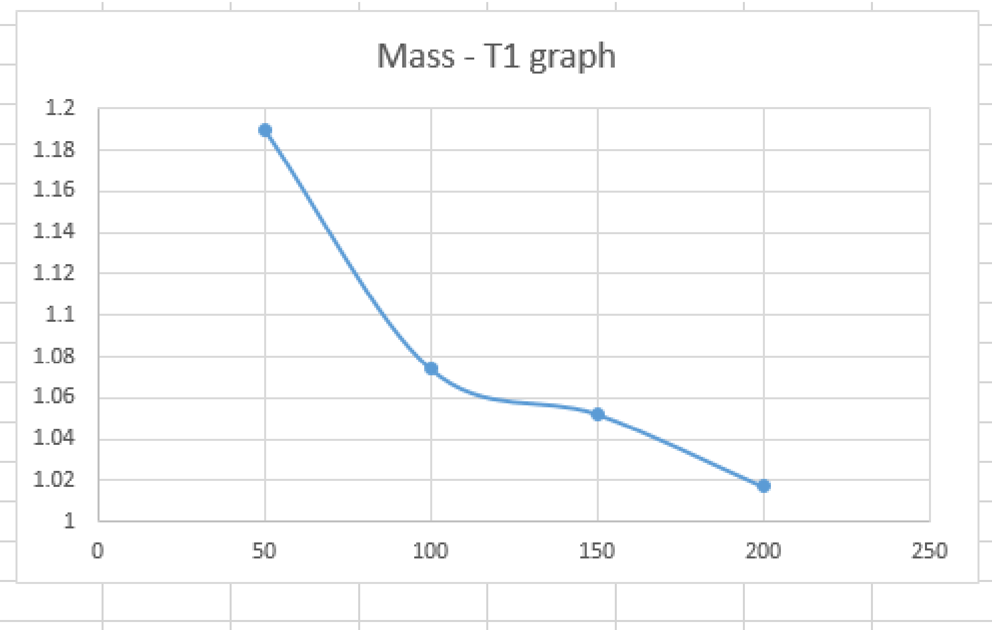


Graph plotted between mass in the centre and time period of pendulum 2

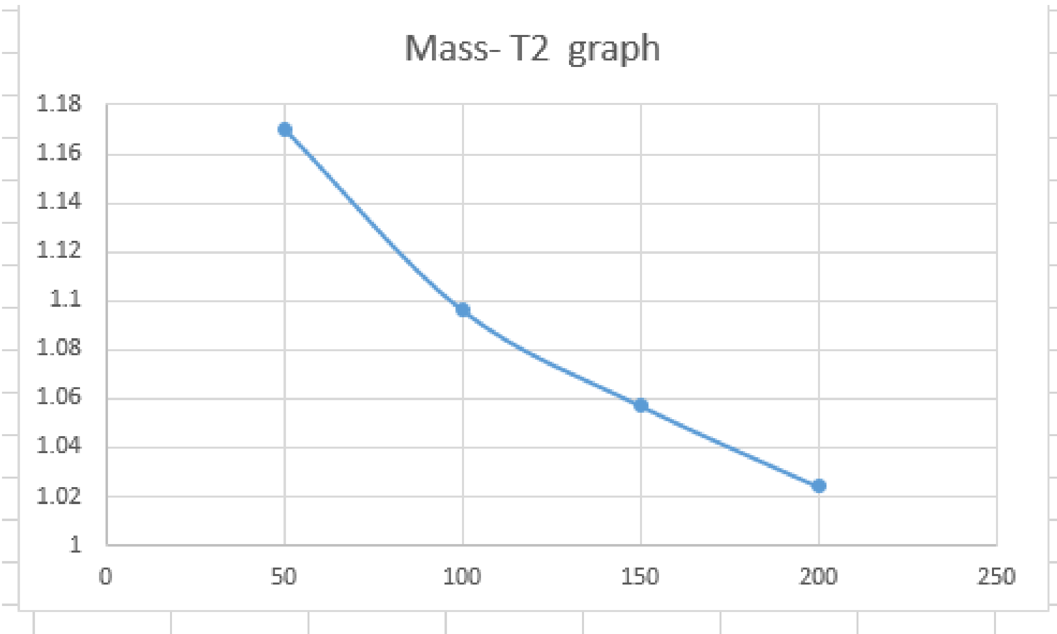


- ***When only one bob is oscillating***

Graph plotted between mass in the centre and time period of pendulum 1



Graph plotted between mass in the centre and time period of pendulum 2



3.3 LISSAJOUS PATTERN

DISCOVERY OF LISSAJOUS PATTERN

Lissajous figure, also called BOWDITCH CURVE, is a pattern produced by the intersection of two sinusoidal curves the axes of which are at right angles to each other. First studied by the American mathematician Nathaniel Bowditch in 1815, the curves were investigated independently by the French mathematician Jules-Antoine Lissajous in 1857–58. Lissajous used a narrow stream of sand pouring from the base of a compound pendulum to produce the curves.

Jules Antoine Lissajous professor of mathematics at the Lycée Saint-Louis in Paris, in 1857 , published "Mémoire sur l'Etude optique des mouvements vibratoires," *Annales de chimie et de physique*, 3rd series, 51 (1857) 147-232, 2 folding plates. Lissajous's paper on his optical method of studying vibration gave rise to the widely used "Lissajous figures," or Lissajous curves, defined mathematically as curves in the xy plane generated by the functions

$$y = asin(\omega_1 + q_1)$$

$$x = asin\,(\omega_2 + q_2)$$

where ω_1 and ω_2 are small integers. Lissajous was interested in demonstrations of vibration that did not depend on the sense of hearing. His most important research, first described in 1855, was the invention of a way to study acoustic vibrations by reflecting a light beam from the vibrating object onto a screen. Lissajous produced two kinds of luminous curves. In the first kind, light is reflected from a tuning fork (to which a small mirror is attached), and then from a large mirror that is rotated rapidly. The second kind of curve, named the 'Lissajous figure,' is more useful. The light beam is successively reflected from mirrors on two forks that are vibrating about mutually perpendicular axes. Persistence of vision causes various curves, whose shapes depend on the relative frequency, phase, and amplitude of the forks' vibrations. If one of the forks is a standard, the form of the curve enables an estimate of the parameters of the other. As Lissajous said, they enable one to study beats (the ellipses rotate as the phase difference changes). 'Lissajous figures' have been, and still are, important in this respect" (DSB).

Lissajous figures are sometimes used in graphic design as logos. Examples include the logos of the Australian Broadcasting Corporation (a = 1, b = 3, d = p/2) and the Lincoln Laboratory at MIT (a = 4, b = 3, d = 0).

Prior to modern computer graphics, Lissajous curves were typically generated using an oscilloscope. Two phase-shifted sinusoid inputs are applied to the oscilloscope in X-Y mode and the phase relationship between the signals is presented as a Lissajous figure. Lissajous curves can also be traced mechanically by means of a harmonograph. They often appear in computer screensavers.

- **LISSAJOUS PATTERN USING COUPLED PENDULUM**

The coupled pendulum can be created with either string or a spring connecting the two pendulums. Here we used a string to connect the pendulum. With each swing energy is transferred from one pendulum to the other. If the pendulums both have the same length one pendulum comes to a complete stop before alternating motion. This phenomenon can be used to show balanced and unbalanced forces, how motion can be used to predict future motion, and the conservation of energy.

Every pendulum has a natural or resonant frequency, which is the number of times it swings back and forth per second. The resonant frequency depends on the pendulum’s length. Longer pendulums have lower frequencies.

Every time the first pendulum swings, it pulls on the connecting string and gives the second pendulum a small tug.

Since the two pendulums have the same length, the pulls of the first pendulum on the second occur exactly at the natural frequency of the second pendulum, so it (the second pendulum) begins to swing too. However, the second pendulum will swing slightly out of phase with the first one. When the first pendulum is at the height of its swing, the second pendulum is still somewhere in the middle of its swing.

As soon as the second pendulum starts to swing, it starts pulling back on the first pendulum. These pulls are timed so that the first pendulum slows down.

To picture this, it may help you to think of a playground swing. When you push on the swing at just the right moments, it goes higher and higher. When you push the swing at just the wrong moments, it slows down and stops. The second pendulum pulls on the first pendulum at just the “wrong” moments.

Eventually, the first pendulum is brought to rest; it has transferred all of its energy to the second pendulum. But now the original situation is exactly reversed, and the first pendulum is in a position to begin stealing energy back from the second. And so it goes, the energy repeatedly switching back and forth until friction and air resistance finally steal all of it away from both pendulums.

If the two pendulums are not the same length, then the tugs from the first pendulum’s swings will not occur at the natural frequency of the second one. The two pendulums swing, but with an uneven, jerky motion.

It’s easy to predict how often the two swinging cans will trade energy. Count the total number of swings per minute when you start both pendulums together and they swing back and forth, side by side.

Compare that to the number of swings per minute when you start them opposite one another—that is, with one pulled forward and one pulled backward an equal distance from the string, and then released at the same time.

The difference between those two numbers exactly equals the number of times per minute that the pendulums pass the energy back and forth if you start just one pendulum while the other hangs at rest.

Physicists call these two particular motions normal modes of the two-pendulum system, and they call the difference between the frequencies of the normal modes a beat frequency.

- **EXPERIMENTAL SETUP**

To study Lissajous pattern, here we suspended a string from the middle of another string whose ends are separated at a particular distance. The strings will give a Y shape. At the end of the suspended string, we tied a sand bottle filled with freely falling sand. The sand falls from a small hole made at the bottom of the hole.

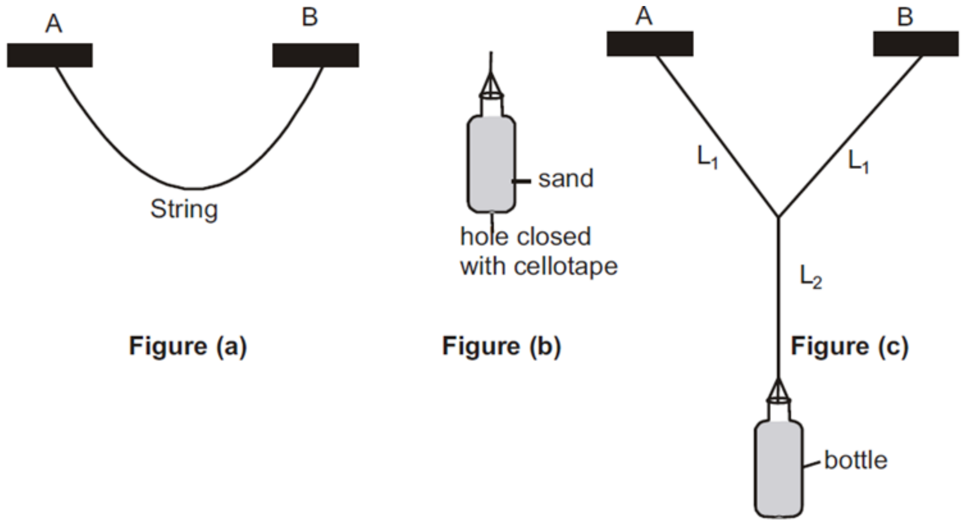


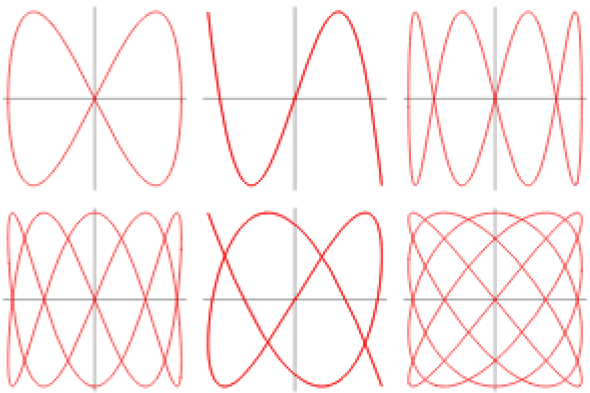
Figure (a) Figure (b) Figure (c)

The pattern we obtain from the falling sand is the matter of our study. The coupled sand pendulum we have here may have two periods. If we oscillate the pendulum perpendicular to the plane, the whole of the string which is L1, L2 and L3 will oscillate. Different from that if the pendulum is subjected to oscillation in the horizontal plane itself it will cause the oscillation of L2 only. Thus we can conclude that a simple pendulum is capable of producing two periods depending on the mode of oscillation. When in a simple pendulum the time period of both horizontal and vertical straight oscillation is recorded to be the same, in a coupled pendulum it is observed as less. The period of the horizontal oscillation of the coupled pendulum is found to be less as when length of the pendulum decreases the period of oscillation also decreases. If a simple pendulum is oscillated in a particular angle we will obtain the pattern of an ellipse spiralling inwards due to the frictional loss of energy of the pendulum. In a coupled pendulum due to the combination of the two periods and the frictional loss of energy we will get an interesting pattern. A Lissajous figure is a pattern which is displayed on the screen when sinusoidal signals are applied to both horizontal & vertical deflection plates of CRO. These are used to measure the frequency of the given signals and phase difference between the signals.

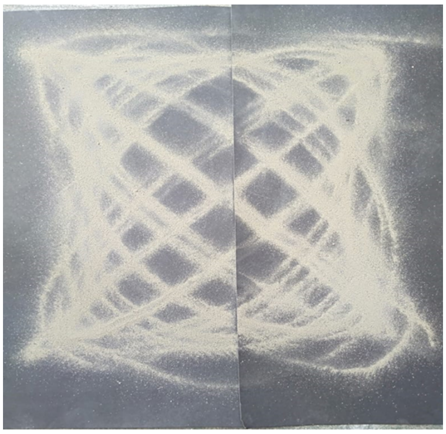
If the frequency and phase angle of the two curves are identical, the resultant is a straight line lying at 45° (and 225°) to the coordinate axes. If one of the curves is 180° out of phase with respect to the other, another straight line is produced lying 90° away from the line produced where the curves are in phase (i.e., at 135° and 315°).

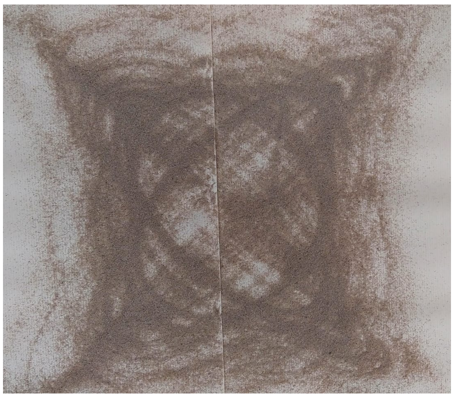
Otherwise, with identical amplitude and frequency but a varying phase relation, ellipses are formed with varying angular positions, except that a phase difference of 90° (or 270°) produces a circle around the origin. If the curves are out of phase and differing in frequency, intricate meshing figures are formed.

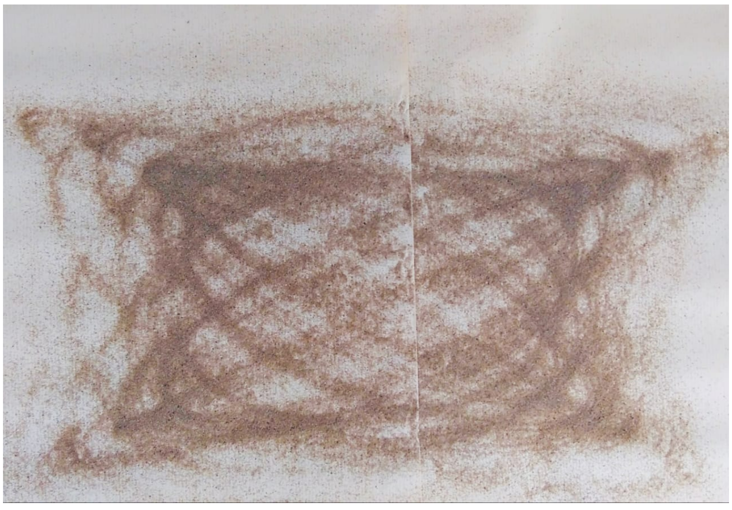
Of particular value in electronics, the curves can be made to appear on an oscilloscope, the shape of the curve serving to identify the characteristics of an unknown electric signal. For this purpose, one of the two curves is a signal of known characteristics. In general, the curves can be used to analyse the properties of any pair of simple harmonic motions that are at right angles to each other.



- **OBTAINED PATTERN**







Different versions of the pattern were obtained when the length of the string and angle was changed.

- **USES OF LISSAJOUS PATTERN**

The Lissajous figure’s uses mainly consist of measurement of the frequency and measurement of the phase difference. The Lissajous figure is of high importance in physics in order to study the sinusoidal waves. The Lissajous figures are mainly used in analogue electronics to analyse the intersection of two or more sinusoidal wave constructing loops which are also known as Lissajous knots in general.

Some of the usage of Lissajous figure are as follows:

- The Lissajou figures are used to determine the unknown frequency by comparing it with the known frequency.
- Verifying audio oscillator with a known-frequency signal.
- Monitoring audio amplifiers and feedback networks for phase shift.

CONCLUSION

From the first part of the project it is clear that the time period for oscillation of the individual pendulums decreases as the mass suspended in the centre increases.This shows the damping effects of the coupled oscillation. As the mass increases more and more is the hindrance offered by the mass to the oscillation of the pendulum. And hence the time of oscillation of the 1 and 3 decreases in addition to mass to pendulum 2. The graph is plotted showing the variation time period to the mass added. It can be observed that at a particular length this variation in the time period disappears. At that particular length the variation of mass has no effect on the time period of the pendulums.

In the second part of the project, various patterns obtained from the sand pendulum are studied. When we oscillate the sand pendulum in the plane of the tied string and in the plane perpendicular to it, we get a straight line as the pattern. On oscillating the sand pendulum in different angles, we get different patterns. The lissajous patterns were obtained and studied using the Sand pendulum.

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