Project Report

On

A BRIEF STUDY ON THE GEOMETRICAL CONCEPTS BEHIND ROBOTICS

Submitted

in partial fulfilment of the requirements for the degree of

BACHELOR OF SCIENCE

in

MATHEMATICS

by

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CERTIFICATE

This is to certify that the dissertation entitled **A BRIEF STUDY ON THE GEOMETRICAL CONCEPTS BEHIND ROBOTICS** is a bonafide record of the work done by **ALINA ROSE P S** under my guidance as partial fulfillment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

Date: 16-02-2023
Place: Ernakulam

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DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of Donna Pinheiro, Assistant Professor, Department of Mathematics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

Place: Ernakulam. **ALINA ROSE P S**

Date: 16-02-2023 **AB20AMAT042**

ACKNOWLEDGEMENT

I want to express my sincere gratitude to Donna Pinhero, Assistant Professor in the Department of Mathematics at St. Teresa's College, who served as our project's coordinator and made our work possible.I was able to complete all of my project's writing stages because of her direction and assistance.

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Finally, I want to express my gratitude to God for guiding me through all of the challenges. Every day, I have felt your guiding.

Place: Ernakulam. **ALINA ROSE P S**

Date: 16-03-2023 **AB20AMAT042**

Contents

INTRODUCTION

Robotics is an interdisciplinary field that has equal importance in computer science and mathematics. A robot might be thought as 'forced labour' because it is made to perform tasks far more quickly than human brains can. The human brain has always been keenly interested in how these robots perform a task that is designed to be done by humans but in a faster way. They can understand and use the concepts of mathematics to solve a task which is the hidden agenda behind every problem solving.

Artificial intelligence is the idea of controlling robots to carry out predetermined instructions. The construction of robots, on the other hand, necessitates the knowledge of geometry because it raises issues regarding sizes, dimensions, shapes, angles, and other factors, demonstrating the interaction between the concepts of robotics and mathematics.

Two-dimensional coordinates, specifically those with two axes, the x and y axes, are the most commonly used expression in the study of geometry. They allow us to locate a figure in a coordinate plane.

The project focuses on the theorems used in robotics, how they may be applied selectively to solve a problem in real life, and how to use computer language to find a mathematical solution to this problem.

Chapter 1

MATHEMATICS BEHIND ROBOTICS

The exponential expansion of robotics research is ushering in a new industrial revolution. Currently, there are more than a million robots in operation worldwide.

Many robots today have brainpower that is equal to or even exceeds that of humans in terms of intelligence, physical ability, perception, and behaviour. In some disciplines, such as computer-assisted surgery, these intelligent machines can even outperform people.

Robots are performing a wide range of crucial tasks for society, including handling potentially hazardous items, weld components, office labour including filing client information, and carpet cleaning.

1.1 APPEARANCE OF ROBOT IN HUMAN HISTORY

Mathematical research predates robotics research in history. The word "robot" is attributed with being popularized by the 1920 play "Rossum's Universal Robots" by Czech author Karel Čapek.

A robot with programming that could sit, rotate, and move its limbs was made in the 18th century by Leonardo da Vinci. In the 19th century, robotics made significant strides because to the efforts of Charles Babbage and Ada Lovelace.

- Modelling ,planning and control of robots using mathematics

Mathematically speaking, a robotic system can be viewed as a com-

plex system made up of operations in numerous subsystems. It is categorized based on its mobility properties and serves as the conceptual bridge between perception and action. A robotic system has more flexibility the more difficult the mathematical concepts are.

To find the right sized parts, the measurements to perform the tasks, to test the performance, or to detect the patterns and relationships between speed and power, or wheel diameter and distance travelled, you need to understand some fundamental mathematical concept.

1.2 METHODOLOGY

The discipline of robotics, which is becoming increasingly important in the modern world, has benefited greatly from the contributions of mathematics in many ways. The term "The interplay between the robotics and Mathematics" served as the fundamental framework from which the research developed to discover a solution to a problem based on a regular basis of a real life situation

In view of the field of mathematics, another application that uses a matrix to rotate the robot's arm and cut the weed is used. The simple application of coordinate theory representation locates the weed from the Hough Transform obtained which is an image processing technique.

As a result, the phrase "Mathematics in Robotics" is proven.

The approaches employed are as follows:-

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

In robotics, the aforementioned matrix with the cosines and sines is frequently referred to as the rotation matrix (or two-dimensional rotation matrix). It aids in the conversion of points or vectors in the local reference frame to points or vectors in the global reference frame.

That's how to describe the rotation of a robot in two dimensions.

Hough Transform

The Hough Transform which is an image processing technique that is used to locate the given weed in the agricultural field where the

points, lines and coordinates are used. The coordinates found from the provided image's end points, as processed by the sensor, constitute the foundation of the Hough Transform's necessary theory.

The weed is then located in the field according to the coordinate system under consideration, and it is cut using the rotation vector that is subsequently applied.

1.3 LITERATURE REVIEW

The oldest branch of mathematics is known as "geometry," which has developed over millennia by using basic objects like points, lines, surfaces, etc. to make contributions to not just the area of mathematics but also to every discipline related to it.

The question of whether or not the great quality of ancient geometry will be able to define or be sufficient to solve many of the problems or duties assigned to robots therefore arises, and it also has equal relevance in this undertaking.

Taking the most significant sense in the human body—vision—as a starting point for the research in relation to the issue of geometry The following research papers, journals, and references were produced as a result of the confluence of these ideas from the group project, such as geometry, vision, and robotics

•*Geometric Reasoning And Artificial Intelligence: Deepak Kapoor's Introduction To The Special Volume* [9] discusses the significance of geometry in robots and their primary component, which is referred to as the "Artificial Intelligence." Being a special volume, it presents fundamental geometric ideas in a formal manner, taking into account current advances in algebraic approaches.

It discusses the history of geometry, from the time of Euclid to the efforts of Gelernter and his colleagues in the 1950s and 1960s to prove the mechanical theorem, which included the first artificial intelligence of the age, using points, lines, and triangles, among other geometrical objects.

•*Robotics And Math Using Action Research To Growth Problems*

[1]: by Shelli L. Casler Failing of Georgia Southern University, which discusses the importance of robots in problem solving in the modern world. It covers issues based on settings and scenarios found in real life, and it proves how a robot—often referred to as "THE MANMADE HUMAN"—solves them using mathematics that has grown significantly over the course of centuries.

It also leads us to the conclusion that robotics is unquestionably necessary for the next generation, who will be more adept at learning and solving mathematical problems, advancing their further education, and ensuring the validity of the curriculum.

• *The Geometric Trajectory Planning For Robotic For The Robot Motion Over A 3d Surface* [5]:The journal work by Bashir Hosseini Jafari, Nolan Walker, Ronald Smaldone, and Nicholas Gans adds to the justification supplied by Shelli L. Caster in the earlier book. It describes how a robot using mathematics' geometrical concepts as its instrument can solve a real-world problem or circumstance.

Mapping a 2D desired pattern to a 3D pattern onto a curved surface is the issue at hand. However, if the challenge is given the requirement that it be mapped in a manner that preserves the length, angle, and area of the original pattern depending on the pattern provided to the machine or robot.

The study continues by naming the different theorems, such as the Equireal mapping theorem, Conformal mapping theorem, and isometric mapping theory. Finally, it comes to the conclusion that modern robots can solve any problem, no matter how vast or challenging it may seem, by applying basic geometric principles.

Chapter 2

MATHEMATICAL THEOREMS IN ROBOTICS

The second chapter covers the fundamental mathematical foundations that must be understood in order to comprehend the project's topic, "Mapping." It covers topics like Quadric Surfaces and the surface parameterizations associated to them, the geometry of matrices, which includes its use in rotation, etc., and progresses in the upcoming Chapter 3.

The mathematical theorems discussed in this chapter speak about the certainty even in machines or robots in maintaining the conditions, such as the length that is to be moved, the angle in which it should spin, etc. based on the mission assigned to it.

2.1 QUADRATIC SURFACES AND SURFACE PARAMETRIZATION

Assume that the pattern that needs to be mapped is a circle with a radius of 3. The circle's equation is then $x^2 + y^2 = 9$ with respect to the circle's center, let's say C. However, in a three-dimensional plane, the given equation also describes a surface. Imagine two copies of the circle placed on top of one another, centered on the z-axis , creating a hollow tube in order to translate one picture of the circle to another. The equation $x^2 + y^2 = 9$ in the xy plane can then be created with a set

of lines that are perpendicular to the z-axis and also cut through the circle. Thus, any of the coordinate planes can be extended to constitute a surface.

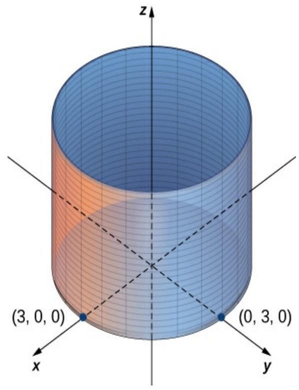

Figure 2.1:

Note:-

• A cylindrical surface, or to be more specific, a "cylinder," is a collection of parallel lines to a given line that traverses a particular curve.

• "The Rulings" refers to these parallel lines.

Finally, it should be noted that when a robot is tasked with mapping a certain 2D pattern as identical as possible but without any distortion or shrinkage, mathematical concepts like parallel lines and other geometric patterns are used.

Since the quadratic surfaces can also be the graphs of equations that can be expressed in the form

$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + K = 0$

Where $A, B, C, D, E, F, G, H,$ **and** I **are the coefficients of the surface equation, which aid in determining the surface's geometric shape, location, and orientation for the purpose of mapping an image to it. We assume the quadratic surface has a standard form for the sake of convenience.**

Example:-

If a robot is instructed to draw an ellipse using the formula $\frac{x^2}{2^2} + \frac{y^2}{3^2} + \frac{z^2}{5^2} = 1$**, the task's solution starts with tracing it using mathematical principles. It set** $z = 0$ **to find the traces in the** xy**plane. This is** $\frac{x^2}{2^2} + \frac{y^2}{3^2} =$

1. **The additional traces are found by setting $x = 0$ and $y = 0$.**

The pattern of the ellipsoid to be mapped is depicted in the appropriate figure below.

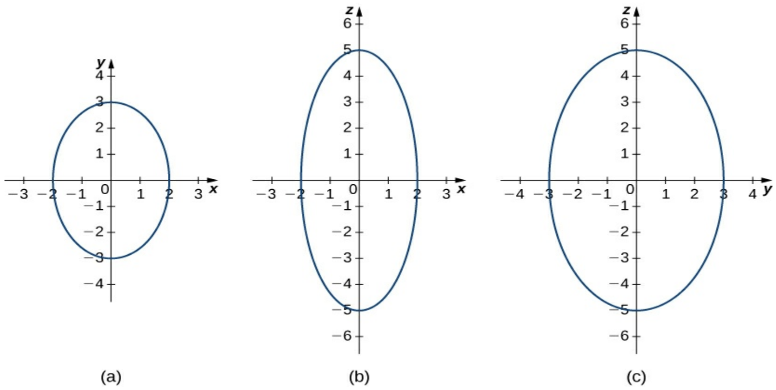

Figure 2.2:

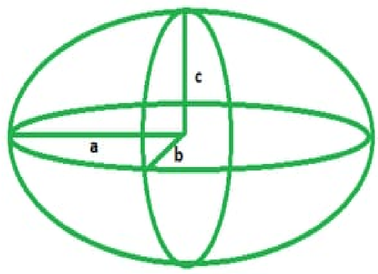

Figure 2.3:

The subject of the lengths of the curves included in the pattern, the angles between the curves, as well as the area of the areas, arises once the pattern's specified geometry has been established.

The robot attempts to answer this question, and its solution is based on the mathematical ideas of curve and surface parameterization.

Let u **and** v **be two variables and we know that** $r_s(u, v) = [x(u, v), y(u, v), z(u, v)]$ **defines a surface parameterization where** r_s **is an element of** R^3 **and** (u, v) **element of** R^2**. Here the surface is defined by** $x(u, v), y(u, v), z(u, v)$**, each of which is a function of two variables** u **and** v**. The image to be mapped is represented by** $r_c(\theta) = [x(\theta), y(\theta), z(\theta)]$ **where** θ **is an element of** R **which is the curve parameterization and is written as** $r_{cp}(\theta) = [u(\theta), v(\theta)]$ **where** r_{cp} **is an element of** R^2**.**

2.2 THE MATRIX OF DIFFERENTIAL GEOMETRY BEHIND THE ROTATIONS

Imagine instructing a robot to rotate through a specific angle along each of the $x, y,$ and z planes when mapping needs to be done. All x, y and z axis are taken into account in three dimensions.

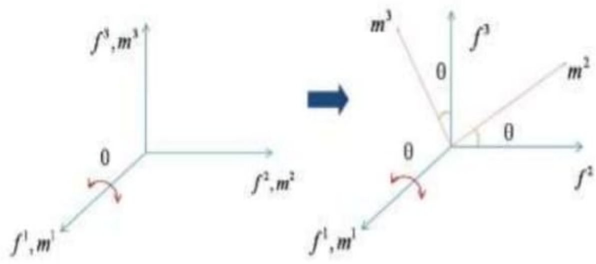

Figure 2.4:

The rotation in the three dimensional plane can be represented by 3×3**.**

$$R_2(\theta) = \begin{bmatrix} f^1m^1 & f^1m^2 & f^1m^3 \\ f^2m^1 & f^2m^2 & f^2m^3 \\ f^3m^1 & f^3m^2 & f^3m^3 \end{bmatrix}$$

Rotation about 1^{st} **axis**

$$R_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix}$$

Rotation about 2^{nd} **axis**

$$R_2(\theta) = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}$$

Rotation about 3^{rd} **axis**

$$R_3(\theta)=\begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

When robots are specifically required to rotate via a certain degree of angle, the usage of a n×n matrix with the appropriate matrix transformations aids the robots in their motion and rotation.

Thus, tackling the mathematically associated physical concepts such as torque, the influence of forces, etc., is also beneficial.

2.3 CONFORMAL MAPPING

Let $w = f(z)$ **be any analytic function in domain** D **of the** z**-plane and let '**z_0**' be an interior point of** D**.**

Let C_1 **and** C_2 **be the two continuous curve passing through '**z_0**' and having definite tangent at this point making angle by** (α_1, α_2) **say with** x**-axis.**

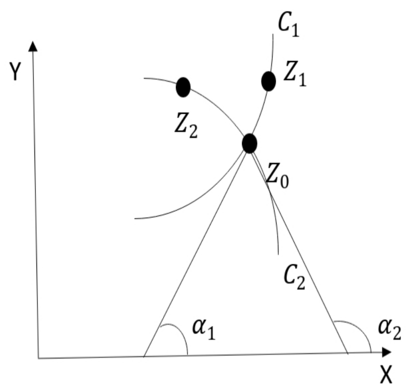

Figure 2.5:

On Z**-plane**

Let C_1' **and** C_2' **be the curve of** W**-plane intersection at** $W_0 = f(z_0)$ **corresponding to the curve** C_1 **and** C_2**intersection.**

Let Z_1 **and** Z_2 **be the point on the curve** C_1 **and** C_2 **near** z_0 **such that,**

$$|Z_1 - Z_0| = |Z_2 - Z_0| = r(say)$$

Then,

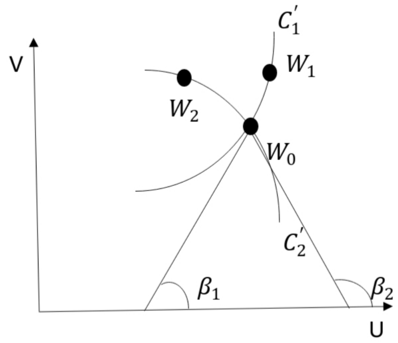

Figure 2.6:

$$Z_1 - Z_0 = re^{i\theta_1} \text{ (mod is changed)}$$

$$Z_2 - Z_0 = re^{i\theta_2} \text{ , where } \theta_1 \to \alpha_1 \text{ as } r \to 0 \text{ and } \theta_2 \to \alpha_2 \text{ as } r \to 0.$$

Let Z_1 and Z_2 correspond to point W_1 and W_2 in the W-plane. Suppose tangent at W_0 to the curves C_1' and C_2' makes angle ϕ_1 and ϕ_2 with real axis.

Let,

$$|W_1 - W_0| = \rho_1 \text{ (say)}$$
$$|W_2 - W_0| = \rho_2 \text{ (say)}$$

Then,

$$W_1 - W_0 = \rho_1 e^{i\theta_1}$$

Similarly, $W_2 - W_0 = \rho_2 e^{i\theta_2}$ **where,** $\theta_1 \to \rho_1$ **as** $\rho_1 \to 0$ **and** $\theta_2 \to \rho_2$ **as** $\rho_2 \to$ ß0.

Now

$$f'(z_0) = \lim_{Z_1 \to Z_0} \frac{f(z_1) - f(z_0}{Z_1 - Z_0}$$
$$f'(z_0) = \lim_{Z_1 \to Z_0} \frac{W_1 - W_0}{Z_1 - Z_0}.$$

That is,

$$f(Z) = W$$

$$f(Z_1) = W_1$$

$$f(Z_0) = W_2$$

That is, $Z_1 \to Z_0$

That is, $Z_1 - Z_0 \to 0$ **and** $Z_1 - Z_0 = r_1 e^{i\theta_1}$

Or, $r_1 e^{i\theta_1} \to 0$

Implies $r_1 \to 0$

Then $\lim_{r\to 0} \frac{\rho_1 e^{i\theta_1}}{re^{i\theta_1}} = \lim_{r\to 0} \frac{\rho_1}{r} e^{i(\phi_1 - \theta_1)}$.

Since $f'(z_0)0$ **, we may write,**

$$f'(z_0) = Re^{ir} \text{ (say)}$$

So that, $f'(z_0) = \lim_{r\to 0} \frac{\rho_1}{r} e^{i(\phi_1 - \theta_1)}$

$$Re^{ir} = \lim_{r\to 0} \frac{\rho_1}{r} e(i(\phi_1 - \theta_1)$$

Equating modulus and amplitude (argument) on both sides, we get that is,

$R = lim_{r\to 0} \frac{\rho_1}{r}$ **and**

$$\lambda = lim_{r\to 0}(\phi_1 - \theta_1)$$
$$\lambda = lim_{r\to 0}\phi_1 - \lim_{r\to 0} \phi_1$$
$$\lambda = \beta_1 - \alpha_1$$

So, $\beta_1 = \lambda + \alpha_1$

Similarly, $\beta_2 = \lambda + \alpha_2$

Thus, $\beta_2 - \beta_1 = \alpha_2 - \alpha_1$**.**

Thus the angle between C_1' **and** C_2' **at** W_0 **is equal in magnitude to the angle between** C_1 **and** C_2 **at** z_0 **has same length.**

Therefore it is conformal.

This preserves the angle between two curves whose lengths are already equal.

In order to maintain the angle formed between the sides of a geometric shape where the sides are in length conservation and a robot is assigned the task of solving the problem, conformal mapping theory is applied.

Example: An Equilateral triangle where the sides are congruent and angles are equal.

2.4 ISOMETRIC MAPPING THEOREM

A mapping from a surface S_1 to another surface S_2 is considered to be isometric if and only if a parameterization $r_s : U \to S_1 \subseteq R^3$ and $r_s^{':U \to S_2 \subseteq R^3}$ with the first fundamental from which is $I_{rs} = I_{r's}$.

Proof:-

Let us consider 'F' to be an isometry. The image of a line segment under 'F' is also a line segment. In fact, the image of the line segment $\overline{\mathrm{PQ}}$ under the line segment of F is the line segment between $F(P)$ and $F(Q)$ where P and Q are the two set of points.

We shall first demonstrate that it holds true for just one point before demonstrating that it holds true for every point..

Let 'X' be a point on the line segment $\overline{\mathrm{PQ}}$.

That is,

$$X' = F(X)$$
$$P' = F(P)$$
$$Q' = F(Q)$$

$d(P, X) = d(P', Q') \to isometry$
$d(P, Q) = d(P', Q') \to isometry$
$d(X, Q) = d(X', Q') \to isometry$

It says that, $d(P, Q) = d(P, X) + d(X, Q)$ if X is on ($\overline{\mathrm{PQ}}$.

Figure 2.7:

$d(P', Q') = d(P', X') + d(X', Q')$ **if X' is on $\overline{\mathrm{PQ}}$ at a distance 'r' from P' and also let us assume that, X on PQ is also at a distance 'r' from P. Then $0 \leq r \leq d(P, Q) = d(P', Q')$.**

Corollary:

Here the line segment was a straight line. That is, "Isometries preserve straight lines and their lengths".

The two points of the image created here maintain the spacing between the two points of the item.

•Thus the property of isomerism preserves the lengths of the lines between the points.

In other words, if a robot is requested to map or see a given figure based on geometry with the requirement that its length be retained, the theory the robot will apply to do the task is of isometric mapping.

Chapter 3

HOUGH TRANSFORM

The notion that serves as the foundation for the application done in the following chapter 4, which is covered in detail in this chapter, is the term "Hough Transform."

The Hough Transform and its advancements are covered in detail in Chapter 3 before transitioning to Chapter 4, where it is claimed that the problem can be solved with Hough Transform with ease.

3.1 HOUGH THEOREM

A method for processing images called the Hough Transform was developed by V.C. A censor may utilize the Hough Transform as one of their feature extraction techniques to give consumers information. In addition, it is a technique that aids in the conversion of coordinates in the Cartesian plane to those in the Polar form. When a machine is performing a task in a field that has been handed to it, it helps us locate a specific feature or species from the area that has been censored by the machine.

Hough, which aids in the system's censored line and point detection. Satellites, image analysis, computer vision, digital image processing, etc. are the principal applications for it. It operates by taking a sensed image provided by the satellite and locating the end edges by drawing lines from the image. In other words, the Hough Transform operates on the principle that every edge with edge points can be transformed into every line that could be used to locate a target on the region where

the satellite has acquired information, making it more effective also in identifying its shape and geometry of the object to be targeted, etc.

3.2 PROPERTIES OF HOUGH TRANSFORM

Mathematically, the Hough Transform (HD) has its own words that should be used. They are:

• Rows: These are the image's distance coordinates.

• Columns: The angle coordinates of the image sensor are referred to as columns.

• Peak Function: The shape, geometry, and location of the target are eventually recognized by the Hough transform, and it is also referred to as the Hough function.

• Accumulator: These are the number of unidentified factors whose value determination aids in determining the target's location.

Most things that have 2D shapes, such as circles, eclipses, lines, etc., require the Hough transform. It is ineffective for detecting targets from censored images when there are difficulties with alignment, etc.

Take a look at the image below as an example. The blue point in the image domain was mapped to the blue line in the Hough domain. The red point remains unchanged. The intersection of the blue and red lines in the Hough domain contains the values m and b of the line $y = mx + b$.

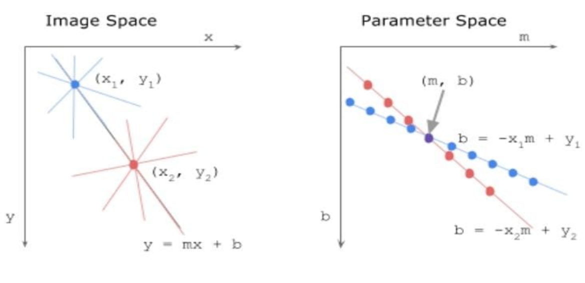

Figure 3.1:

Due to the fact that more points on the same line tend to result in more lines in the Hough domain and that will increase voting to the intersection point, there are numerous points in the picture domain that belong to a line with that slope and y-intercept.

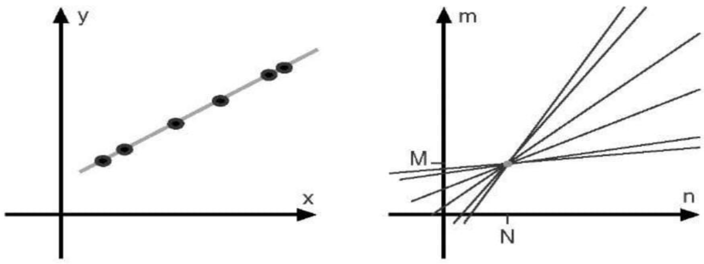

Figure 3.2:

In general, the Hough Transform Theorem may be used to determine the spots, angles, and distance with respect to the limits that are undertaken to the center of the image by inputting a Cartesian coordinate with the aid of two line segments.

Chapter 4

APPLICATION

A nation's development depends in large part on the agricultural sector. And in the contemporary period, when the employment of robots is becoming more significant, robotic technology applications can also be used to find solutions for agricultural tasks like pruning, harvesting, and mowing.

However, the issue of pests must also be taken into mind in order to achieve good crop management. In addition to pest control, numerous other issues like water supply and environmental monitoring need to be addressed. Robots that employ the idea of mapping or localization carry out the task using the satellite signals that are provided to them. And it accurately, dependably, and scalably accomplishes the desired goals.

APPLICATION 1:

A field for farming is nearby. As they develop the strength to handle the pesticides and fertilizers they use on their crops, the farmers in that particular field are having trouble with the overgrowth of weeds on their farm. In order to preserve their crops that have been cultivated alongside them, they must cut or remove the weeds and unneeded plants from the field that is used for farming. They are unable to employ conventional techniques because doing so could result in the complete ruin of the harvest they have grown and, consequently, the farm's reputation for quality. The application of robotic technology that uses geometry and can be used in forestry and agriculture, which is the focus of this

project, is the answer to this problem.

STEP 1

The identification of the weed using the essential fundamental theoretical mathematical stages in the Hough transform image provided by the satellite or sensor.

STEP 2

The robot arm is rotated using a matrix rotation vector after locating the specific row in order to remove the weed.

STEP 3

The steps for the robot to complete its work and produce a meaningful result are input using the required programming language.

4.1 WEED DETECTION USING HOUGH TRANSFORM

Suppose,

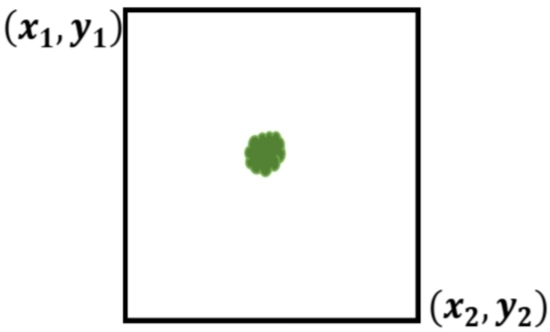

Figure 4.1:

Convert $x_1, y_1, x_2, y_2 \rightarrow x_c, y_c, w, h$

$x_c \rightarrow x$ **center**

$y_c \rightarrow y$ **center**

$w \rightarrow$ *width*

$h \rightarrow$ *height*

In order to find the width and height we must get the center coordinates.

Focusing on one dimension we get, x_c **is between** x_1 **and** x_2**.**

$$x_c = x_1 + \frac{(x_2 - x_1)}{2}$$

Similarly, for y_c

$$y_c = y_1 + \frac{(y_2 - y_1)}{2}$$

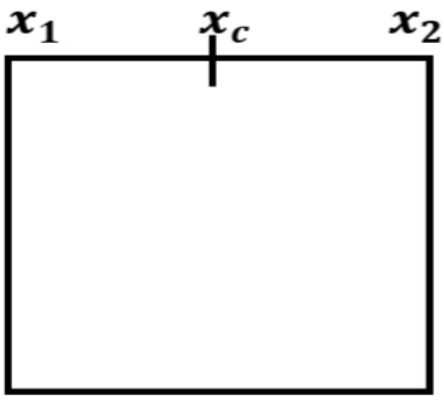

Figure 4.2:

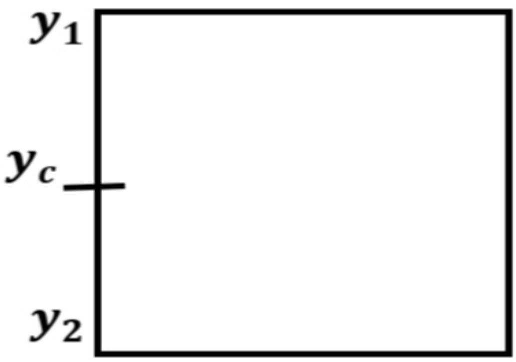

Figure 4.3:

• In the agricultural field below, where the weed is claimed to be in the center, the Hough Transform image processing got a picture of the weed.

1) You are required to locate the center coordinates where the cannabis is allegedly located. For instance,

(The image from the censors)

By using the equation

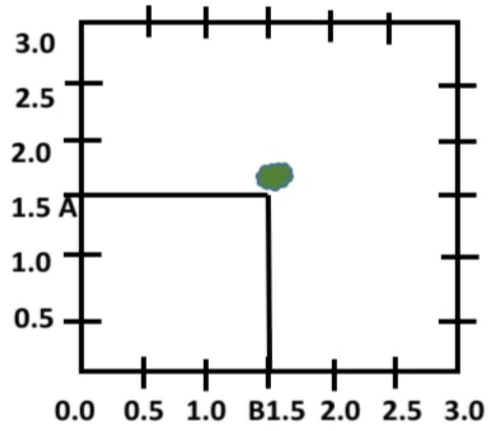

Figure 4.4:

$$x_c = x_1 + |\frac{(x_2-x_1)}{2}| \textbf{ and } y_c = y_1 + |\frac{(y_2-y_1)}{2}|$$

That is let $A(x_1, x_2) - (1.5, 0)$ **and** $B(y_1, y_2) - (0, 1.5)$

That is, $x_c = 1.5 + |\frac{(0-1.5)}{2}|$

$$x_c = |\frac{(1.5+1.5}{2}|$$
$$x_c = 1.5$$

Similarly, $y_c = 0 + |\frac{(1.5-0)}{2}|$

$$y_c = |\frac{1.5}{2}|$$
$$y_c = 0.75$$

That is, coordinates of $C = (1.5, 0.75)$

NOTE:

If the width and height are greater than the specified limit for the plant to be deemed a plant, it becomes a weed; if they are smaller, it is still considered a plant and is treated with disdain.

4.2 SOLVED EXAMPLE USING HOUGH TRANSFORM

Question:-

A)Use the Hough transform to find the locations of weed where (x_1, y_1) **and** (x_2, y_2) **are, respectively,** $(2, 0)$ **and** $(0, 2)$**.**

B)Use the Hough transform to find the locations of weed where (x_1, y_1) and (x_2, y_2) are, respectively, $(2, 0)$ and $(0, 2)$. Check to see if the new location of the robot arm and the position of the weed are the same with respect to the end vertices of robot arm PQ where $P(0, 0)$ and $Q(1, -1)$ with respect to its new origin by using the position of the weed from instance 1 and assuming the length of the robot's arm as $\sqrt{2}$ unit.

Answers

By using the equation of Hough transform, we can detect the position of weed.

$$x_c = x_1 + |\tfrac{(x_2 - x_1)}{2}|$$
$$y_c = y_1 + |\tfrac{(y_2 - y_1)}{2}|$$

Substituting in the equation

$$x_c = 2 + |\tfrac{(0-2)}{2}|$$
$$x_c = 2 + |\tfrac{2}{2}|$$
$$x_c = 2 + 1$$
$$x_c = 3$$
$$y_c = 0 + |\tfrac{(2-0)}{2}|$$
$$y_c = 0 + |\tfrac{2}{2}|$$
$$y_c = 0 + 1$$
$$y_c = 1$$

Therefor the coordinates of weed is $(3, 1)$.

B)Given that the robot's arm is $\sqrt{2}$ unit in length. We deduced that the weed is located at $(3, 1)$ from (A). The robot recognizes the point $(3, 1)$ as being a component of the circle with radius $\sqrt{2}$.

The robot base's new location is in the circle's center. Let this be the case $(2, 0)$.

The base of the robot is always seen as $(0, 0)$. So, in relation to the previous axis, we build a new one.

The weed will now be spotted at the new origin $(2, 0)$, which is considered to be $(0, 0)$, $(1, 1)$.

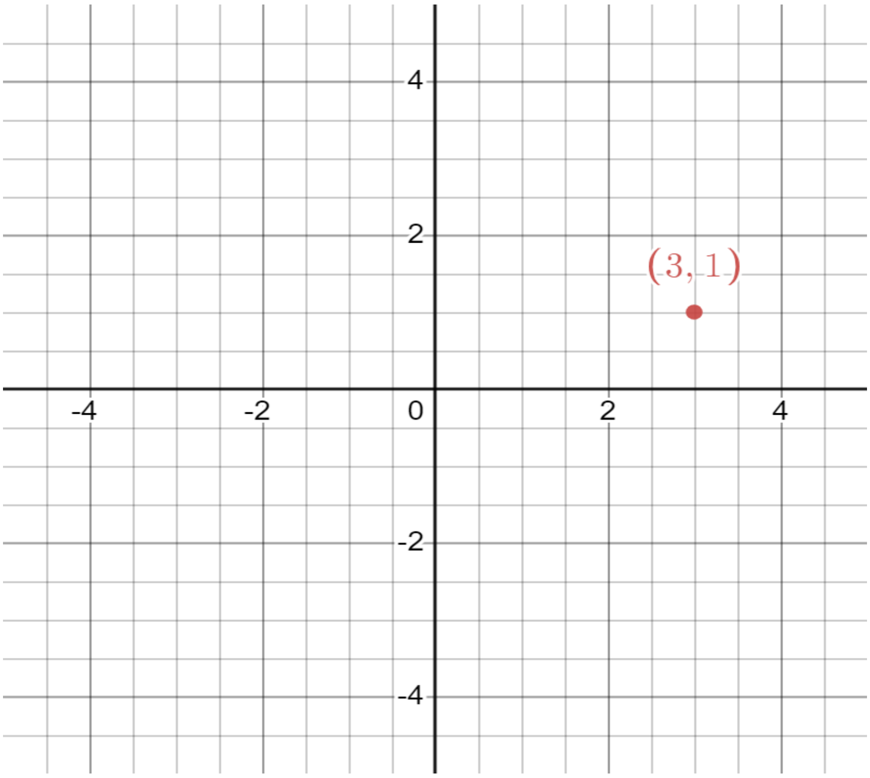

Figure 4.5:

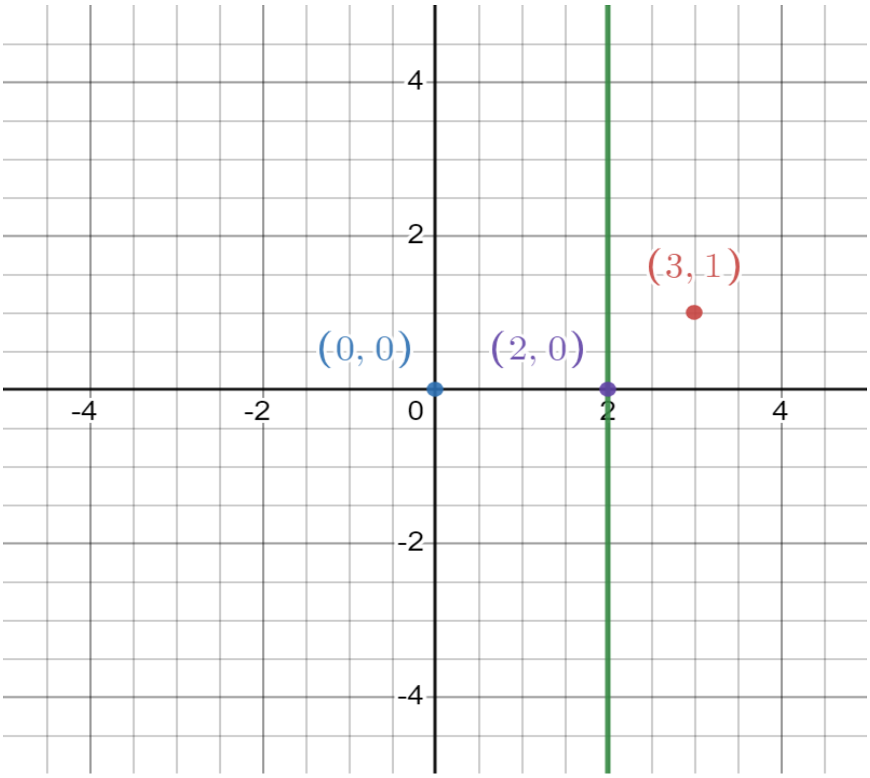

Figure 4.6:

Any resting posture for the robot's arm is possible. The robot will determine the degree of rotation needed to cut the weed based on its current position, its newly discovered coordinates with respect to the new origin, and its detected new coordinates.

Here let the resting position of the robot's arm be $(1, -1)$**.**

In this case $\theta = 90°$**.**

Substituting in the matrix equation;

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos90° & -sin90° \\ sin90° & cos90° \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$(1, 1)$ **is the required coordinates of the new position of arm.**

It shows that the position of the weed is same as the position of the arm through which weed can be eradicated using the blade.

4.3 PYTHON CODE FOR DETECTING THE COORDINATES OF WEED

```
x1, y1 = [float(x) for x in input('Enter x1,y1\t:').split()]
x2, y2 = [float(x) for x in input('Enter x2,y2\t:').split()]

xc = x1 + abs(x2-x1)/2
yc = y1 + abs(y2-y1)/2

print("Center point (xc,yc) is ({},{})".format(xc,yc))
```

4.4 PYTHON CODE FOR ROTATION USING MATRIX

```
import numpy as np
import math

x1, y1 = [float(x) for x in input('Enter x1,y1\t:').split()]
angle = float(input("Enter angle of rotation\t:"))
angle = math.radians(angle)
sin1 = math.sin(angle)
cos1 = math.cos(angle)
mat1 = [
    [cos1 ,-sin1],
    [sin1 , cos1]
    ]
mat2 =[x1,y1]

result = np.dot(mat1,mat2)
print(result)
```

CONCLUSION

We are aiming to explain the mathematics underlying robots working in agricultural fields with this project. To locate the weed and determine how to eradicate it from the field, we have employed a number of techniques, such as the 2D rotation of robots and the Hough Transform. In order to reduce the amount of labour required on this account, the project also includes Python computer code for weed detection.

Thus, it clarifies how mathematics is important to robotics. These steps can be used wherever that requires mathematical comprehension and analysis, not just in robots. Theorems that are solely significant in mathematics and studies related to it have a full explanation of how they fit into the interaction between mathematics and robotics.

The goal of this project was to find a solution in a step more advanced mathematical version in the sector of agriculture. The field of mathematics, which has grown over the years by also providing its own new endeavours, has been relevant in many different areas of this project.

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