Project Report

On

TRIGONOMETRY AND ITS APPLICATIONS

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by

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CERTIFICATE

This is to certify that the dissertation entitled, **TRIGONOMETRY AND ITS APPLICATIONS** is a bonafide record of the work done by Ms. **NANDANA D.V** under my guidance as partial fulfillment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

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DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of Dr.Susan Mathew Panakkal, Assistant Professor, Department of Mathematics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

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I must mention several individuals who encouraged me to carry this work. Their continuous invaluable knowledgably guidance throughout the course of this study helped me to complete the work up to this stage

I am very grateful to our project guide Dr. Susan Mathew Panakkal for her guidance and support throughout the work. I also want to express my gratitude to my classmates, who assisted me in formulating my research topics by sharing their knowledge. Last but not the least, I would like to express my heartfelt gratitude to God Almighty for always being my emotional rock and teaching me how to confront every challenge with intelligence and confidence.

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Chapter 1

INTRODUCTION

Trigonometry is a branch of mathematics that deals with the relationship between the sides and angles of the triangle. The word trigonometry is the combination of two words trigono and metron. In Greek, trigon means triangle and metron means to measure. The basics of trigonometry deal with problems that are related to angles and the measurement of angles. There are six trigonometric ratios and they are sine, cosine, tangent, cosecant, secant, and cotangent. All the important topics covered under trigonometry are based on these trigonometric ratios. The application of trigonometry can be related to different fields of our day-to-day lives and also plays a major role in astronomy where trigonometry helps to find the distance of the earth from planets and stars. It is also used for constructing maps in geography and navigation. The application of trigonometry plays a major role in criminology and marine biology. Based on the concept of trigonometry some of the advanced methods are used in engineering and physical science. In this project, we deal with the important applications of trigonometry namely sound waves, bloodstain analysis, and oceanography.

1.1 HISTORY

The computation of absolute values from the absent absolute value of a portion of the triangle was chiefly concerned with trigonometry. When we look back at the ancient history of trigonometry, we remember the names of these people Pythagoras, Aristotle, and the ancient civiliza-

tion of the Greeks Egyptians, and Babylonians. The author of the Pythagorean theorem is Pythagoras and its states that the square of the length of the hypotenuse is equal to the sum of squares of the length of the other two sides of the right -angled triangle. The table of values for trigonometric functions was formulated by Hipparchus in 120 BCE. He was also familiar with the basic formulas of plane trigonometry. In the 17th century, the symbols for trigonometry were formulated. Ptolemy's Almagest, the first major work of ancient trigonometry to reach Europe did use elementary trigonometry. which lead to Ptolemy's geocentric system which was the succession of the heliocentric system of Nicolas Copernicus. Before Pythagoras was born Babylonians used the Pythagorean theorem. They wrote it down and now it is known as Plimpton 322. The major contributors to trigonometry include India and the Islamic world which makes you wonder about the importance and value of mathematics.

1.2 PRELIMINARIES

Sinusoidal waves -A sine wave or sinusoidal wave is the most natural representation of how many things in a nature change state.

Transverse wave-A wave in which the medium vibrates at right angles to the direction of its propagation.

Longitudinal waves-A wave vibrating in the direction of propagation.

Density- The quantity of something per unit volume, unit area, or unit length

Amplitude- The maximum displacement or distance moved by a point on a vibrating body or wave measured from its equilibrium position.

Pitch-The property of a sound and especially a musical tone that is determined by the frequency of the waves producing it.

Wavelength- The distance between successive crests of a wave, especially points in a sound wave or electromagnetic wave.

Frequency-The number of waves that pass a fixed point in unit time.

Angular Frequency-The angular displacement of any wave element

per unit of time or the rate of change of the waveform phase.

Ambient sounds- The background or surrounding noise.

The angle of impact-The acute angle formed between the direction of a blood drop and the plane of the surface it strikes.

Projectile- A body projected by external force and continuing in motion by its own inertia.

Tides -They are very long-period waves that move through the ocean in response to the forces exerted by the moon and sun.

Seafarers - The person who is employed to serve aboard any type of marine vessel.

Voyages - a long journey involving travel by sea.

Trade winds -They are winds that reliably blow east to west just north and south of the equator.

Vikings - Warriors from northern Europe.

Pillage - rob a place using violence, especially in wartime.

Expeditions - a journey especially by a group of people for a specific purpose

1.3 SOME APPLICATIONS OF TRIG0NOMETRY

Application of trigonometry is applied in many areas such as architecture, celestial, mechanics, surveying, etc. The commonly used field is astronomy and physics which helps to find the distance, analyzing the waves and the path in motion.

It also helps to find the distance of the long river and also helps to measure the height of the mountain etc.

It also plays a major role in various fields like oceanography, seismology, electronics, navigation, astronomy, physical science, and many more.

Spherical trigonometry is used for lunar, locating solar and stellar positions.

Here we are going to focus on three applications of trigonometry which are bloodstain pattern analysis, sound waves, and oceanography. Let's look at the chapters briefly.

Chapter 2

BLOOD PATTERN ANALYSIS

Bloodstain patterns analysis is a forensic discipline that deals with the physics of the blood and assesses bloodstains left at crime scenes using visual pattern recognition.

2.1 TYPES OF BLOOD STAINS ,PRINCIPLES AND SURFACE TEXTURE

Basically, there are 3 types of blood stains. Passive stains, transfer stains, and impact stains.

Passive stains:-

Passive stains are stains created by the force of gravity. These stains occur with indirect forces such as gravity acting on an injured body.

Figure 2.1: figure of passive stains

Transfer stains:-

A bloodstain results in contact between a blood-bearing surface and another surface. These can be drag marks, cloth patterns, shoe or footprints, wipes, and swipes.

Impact blood stain spatter are smaller than passive blood stains. Impact blood spatter is the most common blood stain pattern type at

Figure 2.2: figures of transfer stains

a crime scene. In impact blood stain spatter patterns, blood is often circular and not elongated.

There are 2 types of impact spatter back spatter and forward spatter.

Figure 2.3: figure of impact blood spatter analysis

2.2 PRINCIPLES OF BLOOD STAIN PATTERN ANALYSIS

A liquid only takes the form of a drop when it drops from an object. Once the drop is air born, it takes its minimal surface area and in moves in the flight of a spherical shape. Various physical forces such as air resistance and gravity on it.

Gravity is responsible for the appearance of a number of patterns. It causes the so-called flow patterns. Flow patterns are formed by the movement of blood across a surface due to the influence of gravity. Large pools of blood in solid surfaces lead to serum separation. This means that the blood corpuscles sink because of their weight and the serum remains on the surface as a transparent liquid.

EFFECTS OF SURFACE TEXTURE

Rough surface results in irregularly shaped stains with serrated edges and satellite spatter eg: glass, hardwood flooring, carpet, tile, cloth, and paper.

Increasing roughness of an impacted surface increases the number of satellite stains and also, Increasing drop height leads to an increased no of satellite stains.

2.3 ANGLE OF IMPACT IN BLOODSTAINS

In Forensic science, the angle of impact is the angle at Which a blood droplet impacts a surface. By Measuring the width and length of the stain, the angle of impact can be calculated, helping investigators determine the actions that may have taken place at the scene.

Determination of the angle of impact of a blood stain

Formula to measure an impact of a blood stain

Angle of impact , $\theta = sin^{-1}(width/length)$

As the angle of impact is made smaller or more acute, the blood stain pattern will become more elongated, elliptical, or oval in shape.

Eg:- Q) If a blood stain has a width of 8mm and a Length of 20 mm find its angle of impact.

Ans ; Width ,W=8mm

Length , L = 20 mm

By applying the formula, $\theta = Sin^{-1}(width/length)$

$Sin^{-1}(8/20)$= **23.5 °**

2.4 CASE STUDY

2.4.1 Murder case of Clarence hiller

On September 19, 1910, at 2 am Clarence Hiller heard the screams of his wife and daughter in their home in Chicago and he woke up. After a spate of robberies, residents of this South Side neighborhood were already on the edge. Hiller a railroad clerk and the robber Thomas Jennings clashed with each other. They fell down the staircase. Clarence's daughter recalled hearing three shots, followed by her mother screaming upstairs. Neighbors came running but the man had fled the home, leaving a dying Hiller by his front door. After the inquiry by the cop, they found Jennings's coat and through the blood pattern, they find that it was a gunshot.

Figure 2.4: An article from the daily newspaper of chicago

The blood pattern differs in different cases. In this case, the police analyze that the victim was murdered by a gunshot through the blood pattern that spread on the floor and walls.

These are the blood spatter identified by cops from the crime scene. Here we can identify that this is a blood spatter from a gunshot through mathematics.

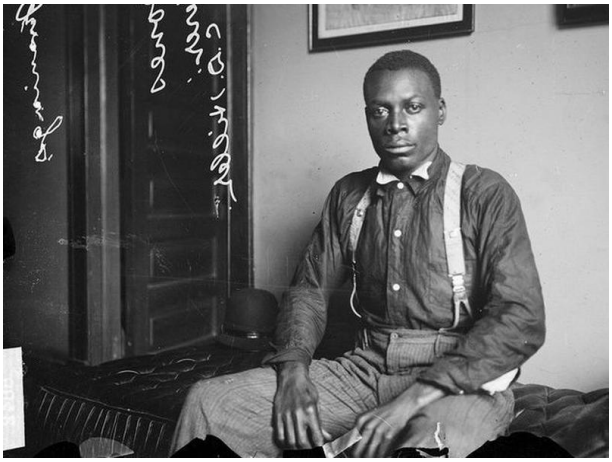


Figure 2.5: Picture of Thomas jennings

Generally in a gunshot, the stain shape varies from circular to elliptical with tails or spines extending in the direction of travel. We get the length and width of the small blood drop hitting the floor which is elongated, as 20mm and 8 mm Hence the angle of impact of the blood stain $= \theta = sin^{-1}(8/20)$**=23.6 degrees.**

Below 75 degrees, spines become more prominent on the side of the spatter opposite the angle of impact. As the angle of impact decreases the spattered stain elongates becoming more elliptical, spines, etc. In a gunshot, the blood stain elongates and it is more elliptical. We get the angle of impact less than 75 degrees. So the victim has been murdered by gunshot. The curved path of objects in projectile motion was shown by Galileo to be a straight line in the special case when it is thrown directly upwards. Such a trajectory is a ballistic trajectory. Internal ballistic deals with the interaction of the gun, and projectile, before the emergence of the projectile from the muzzle of a gun. It is more accurate. The bullet identified from the crime scene is an internal ballistic bullet.

Conclusion: The killer uses internal ballistics as a weapon for murder.

Chapter 3

SOUND WAVES

3.1 INTRODUCTION TO SOUND WAVES

One of the natural trigonometric functions existing in our nature is sinusoidal waves.

An example of it is sound waves.

What is a sound?

In physics, a mechanical disturbance from a state of equilibrium propagates through an elastic material medium.

But simply we can say that sound is something that you can hear.

What are sound waves?

The pattern of disturbance caused by the change in the movement of energy while traveling through a medium (any medium (eg: water, air, etc) which propagates from the source of the sound is called a sound wave.

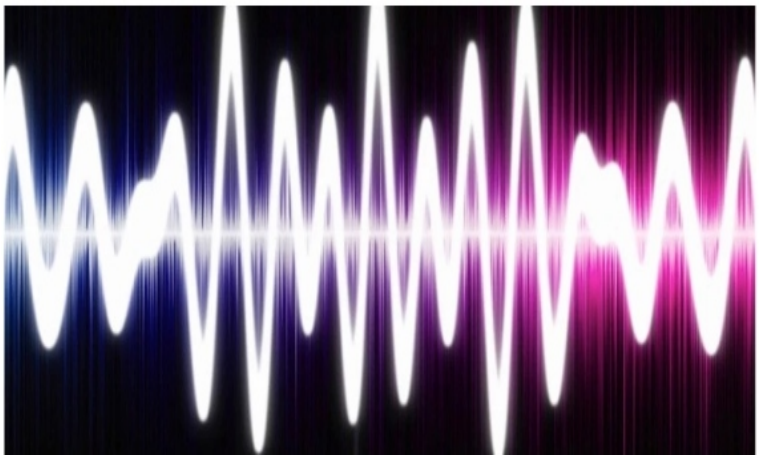


Figure 3.1: Pictorial representation of sound waves

Sound waves are classically mechanical waves. Then what are mechanical waves?

Mechanical waves require a medium for traveling. There are two types of mechanical waves

1) Transverse wave

2) Longitudinal wave A sound wave is a longitudinal wave.

Longitudinal wave

This type of sound wave travels in the same direction according to the movement of the wave.

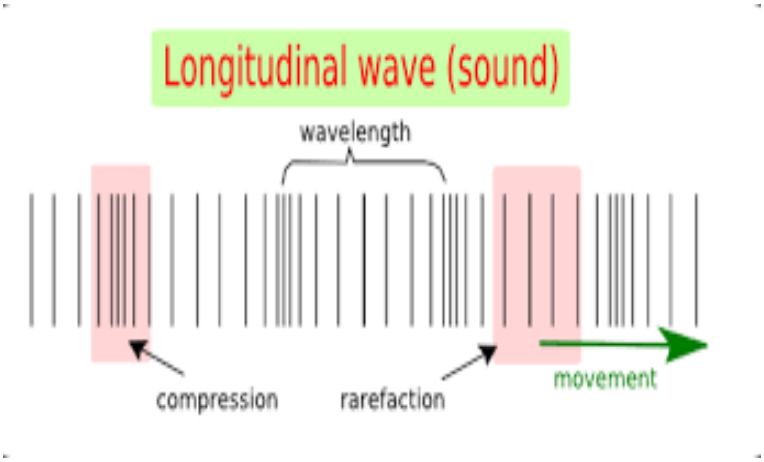


Figure 3.2: Pictorial representation of longitudinal waves

IMPORTANCE OF SOUND WAVES

Sound waves are travelling throughout the atmosphere. Sound waves formed as a result of vibrations of atoms and run into one another. These vibrations occur from a source and travel throughout the atmosphere, the vibrations creating waves of energy. Sound waves are not only to communicate but also to perform various tasks.

COMMUNICATION

Without the help of sound waves, people could not be communicated audibly. Vocal cords help to generate sound waves and then is transmitted through the air to the ears of human beings. Radios and televisions use the same basic concept to transmit sound.

UNDERGROUND RESOURCES

Geologists use sound waves to search for resources. They recoil sound waves into the ground and measure the way in which they travel

through the earth. Geologists can make inferences about the density and makeup of the ground. Geologists can also use the waves that study the possibility of earthquakes.

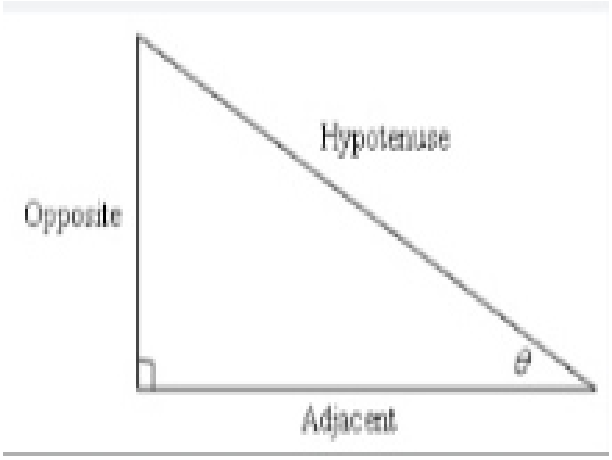
HUNTING

Many creatures use sound waves to hunt for food. Bats usually found prey by using a form of sonar. Bats Project sound waves that bounce off of prey. Bats determine the distance from their prey when the sound waves return back.

3.2 RELATION BETWEEN TRIGONOMETRY AND SOUND WAVES

TRIGONOMETRIC FUNCTIONS

The main functions of an angle of a triangle are called trigonometric functions.



Sinθ= Opposite side/Hypotenuse side

Cos θ = Adjacent side/Hypotenuse side

Tanθ = Opposite side/ Adjacent side

But how these functions are connected to sound waves?

When you listen to music on the radio or hear something on your phone, accordingly your brain is interpreting sound waves that travel through the air.

And we can call these sound waves sine waves. If the sound produced is louder, then the amplitude of the sine curve will be larger. The sound with a high pitch varies with a shorter wavelength and higher frequency.

Frequency and wavelength are indirectly proportional. Frequency can be defined as the number of vibrations per second and its unit is hertz. In this chapter, we are going to investigate the mathematics behind the music which is the relation between trigonometry and sound waves.

3.3 FREQUENCY OF THE NOTES IN A PIANO

Let's start our investigation by understanding the variations in frequency while playing the piano.

Octave: The doubling of frequencies is called octaves. For example, if the frequency is 200 Hz then the octave has doubled the frequency that is 400 Hz. An octave is divided into twelve notes, seven of which are C, D, E, F, G, A, and B; the white keys on a piano. Some countries use other names for the notes. The frequency of the 12 notes is in geometric progression. The ratio of the frequency between two consecutive notes will be constant. Starting from one note to another we multiply each term with the same factor. Let the factor denoted be 'x'.After 12 such multiplications, we get the required doubled frequency.

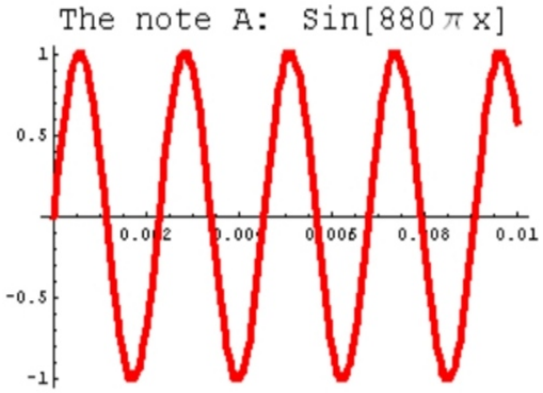
Let's denote this by $x^{12} = 2$

Then we get, x= $2^{1/12}$

Note A (A4) has the frequency f==440 Hz, which means that it has the angular frequency,

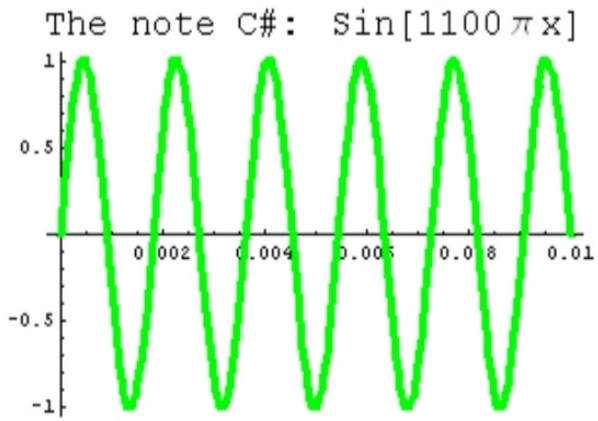
$\omega = 440 \times 2\pi \times x$ **rad/s**

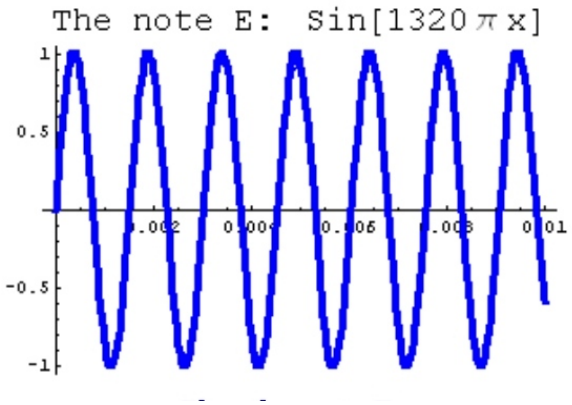
Using this equation we can find out the double frequency which is the angular frequency.



From this, we can conclude that if we change the period of the sine wave then it will change the sound also.

Equation and graphs of the other two notes.





More than one note at a time how is this possible?

In short words, how does adding notes work out?

For this, we have to take all the sine functions for all of the notes and add them together.

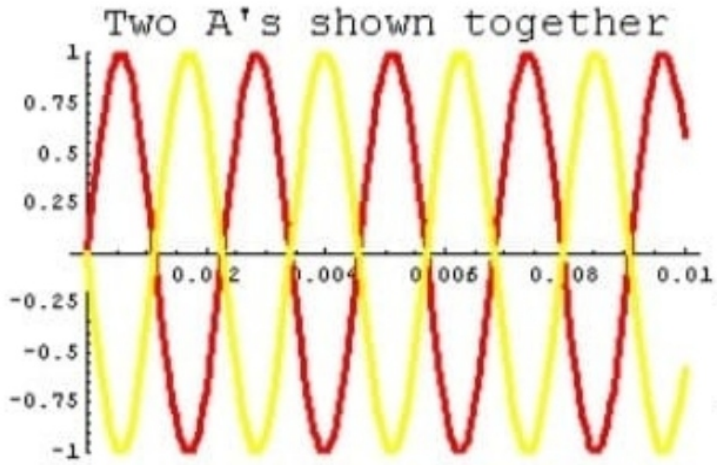
For example, if we add notes A, C, and E, we get a graph like this.

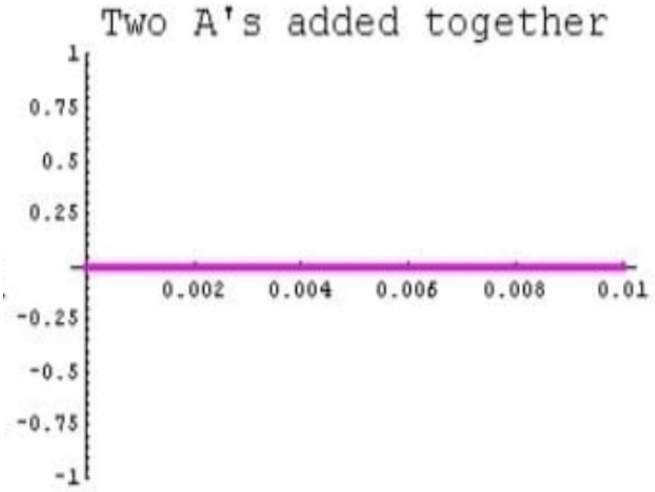
The below graph is no longer a sine curve but has the same pattern which defines the periodic function(figure 3.3)

3.4 CANCELLING SOUNDS

We all have heard about noise-reducing headphones. The actual working principle behind this also is mathematics. First, this kind of headphone finds the sounds reaching your ear. Then it will figure out the

sine waves of ambient sounds. Then it creates additional noise whose sine waves cancel out the ambient sounds Now we can work out one example of this type. Suppose your neighbor keeps playing a recording of the original A note, and it's driving you nuts. It turns out the shifted version above is just the thing to prevent a headache. Below you can see these two sounds graphed together; that picture should help you believe that, when you add these two sine waves together, they cancel, and you get zero.





So from this module, we have understood the role of mathematics in music. In this chapter, we have analyzed each note with the help

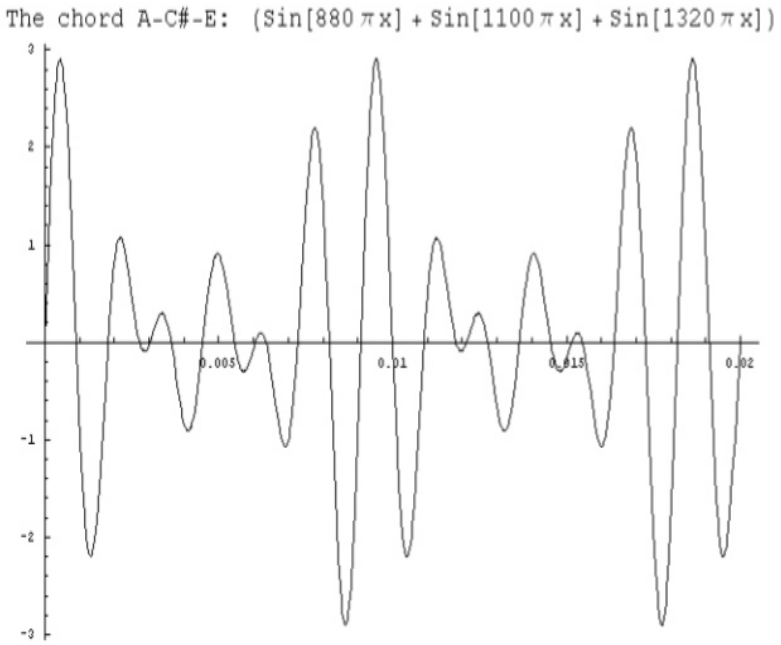


Figure 3.3: Graph of combined notes

of a sine graph. So let's conclude this module by understanding the statement that the "universe is made up of mathematics "from this module.

Chapter 4

OCEANOGRAPHY

4.1 INTRODUCTION TO OCEANOGRAPHY

Oceanography is the study of the physical and biological aspects of the ocean. Oceanography covers a wide range of topics including marine life and ecosystems, ocean circulation, waves, plate tectonics and the geology of the sea floor, and the chemical and physical properties of the ocean. To understand clearly the ocean and how it works, one must know more about the field of science behind it. Oceanography is the combination of various sciences. The objective of oceanography is to increase human understanding of all aspects of the world's oceans and of their processes. Branches of oceanography in different fields

. Chemical Oceanography -Chemical reactions that occur both in the ocean and on the sea floor.

Biological Oceanography Biological oceanography deals with the distribution and environmental aspects of life in the ocean.

Physical Oceanography -Physical reactions such as changes and movement of the ocean.

Geological Oceanography - Geological oceanography is used to study the sediments and topography of the ocean floor.

Ocean Engineering - Ocean engineering is concerned with the development of technology for oceanographic research and exploitation.

Marine Policy -Marine policy considers the application of social and political sciences such as economics, laws, and policy towards the use

and management of the ocean.

Satellite Oceanography - It deals with the measurements of the ocean color, which can be used in assessments of phytoplankton biomass and are of great interest to marine biologists.

4.2 HISTORY OF OCEANOGRAPHY

The knowledge of the waves and currents of the seas and oceans paved the way for the study of oceanography. Observations on tides were recorded by Aristotle and Strabo. Early oceanography began because people wanted to understand how their boats would fare at sea. In fact, understanding the wind and the sea helped some seafarers win several coastal wars. Such knowledge was sometimes referred to as "environmental intelligence." The first evidence we have of voyaging or traveling on the ocean for a specific purpose comes from the records of trade on the Mediterranean Sea. As they went out, mariners began to record information to make their voyages easier and safer. They recorded the location of rocks in the harbor, landmarks, sailing times, and currents. The mariners who made these charts were called cartographers.

The history of oceanography can also be studied by looking at different coastal people and studying how they interacted with the surrounding oceans and seas.

The Chinese built an extensive system of inland waterways. Curiosity and commerce encouraged adventurous people to undertake voyages that were more ambitious.

The Vikings had strong fast ships that allowed them to pillage the coast of Europe quickly. The Portuguese used the "trade winds" to carry their trades along the coast of Africa and India. The Polynesians spread their culture over the Pacific using the prevailing winds but found it difficult to return due to the same winds. There is also a great deal of historic oceanography recorded as more scientific and economic expeditions were launched.

4.3 TRIGONOMETRY BEHIND OCEANOGRAPHY

An oceanographer uses trigonometry for finding the distance. For example, if the oceanographer was tracking a sea creature and they needed to know how far they were from the animal they would use trigonometry

to find out the distance between them and the animal. Oceanographers also use trigonometry to calculate the height of tides in oceans.

4.3.1 Trigonometric ratios

$\angle A$	0^o	30^o	45^o	60^o	90^o
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
cot A	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
cosec A	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Trigonometric ratios are the ratios of the length of sides of a triangle.

These ratios in trigonometry relate the ratio of sides of a right triangle to the respective angle. The basic trigonometric ratios are sin, cos, and tan, namely sine, cosine, and tangent ratios. The other important trig ratios, cosec, sec, and cot, can be derived using the sin, cos, and tan respectively.

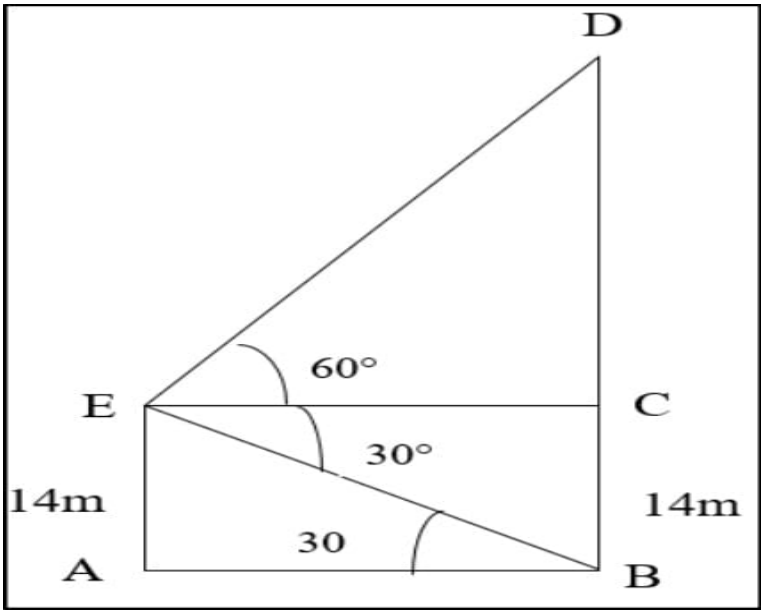
Let's solve a problem using trigonometry which is related to oceanography.

Q) A Man on the deck of a ship, 14 km above the water level, observes that the angle of elevation of the top of a cliff is 60 degrees and the angle of depression of the base of the cliff is 30 degrees. Find the height of the cliff.

Ans:

Given that triangle ABE =30° and DEC =60°

In the right-angled triangle ABE, Tan 30° = AE/AB



AB $=AE\times$**Tan 30°**

AB $=14\sqrt{3}$

EC $=14\sqrt{3}$ **(AB=EC)**

In right-angled DEC, Tan 60° = CD/EC

CD $=EC\times$ **Tan 60°**

CD $= (14\sqrt{3}) \times \sqrt{3}=$ **42 m**

Thus, the height of the cliff, BD = BC+CD = 14+42 = 56 m.

The height of the cliff is 56 m.

From this chapter also it is clear that trigonometry plays a vital role in oceanography.

Chapter 5

CONCLUSION

As we all know that the world is incomplete without mathematics. Our study also proves it. Our study is focused on one of the branches of mathematics that focuses on special functions of angles and their applications the calculation, trigonometry, and its applications in various fields. Trigonometry is associated with sound waves, games, bloodstains and One of the key pieces of evidence in cases such as murder and robbery etc. Furthermore, only through mathematical interpretation can findings be determined such as the height at which the blood drop is generated, or the angle at which the blood drop hits the target surface. The probability that the blood types of any two randomly selected individuals match each other and more. Trigonometry is used to measure the level or pitch of a sound wave or frequency. The frequency is measured in Hertz. The frequency of a sound or light wave is the number of complete waves in a given period of time. Trigonometry also plays an important role in oceanography also. For measuring the distance in oceanography we make use of trigonometric ratios. So the mathematical application of trigonometry is in our daily life without our knowing it. We still use it for many of our inventions and solutions to problems

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