THERMODYNAMIC PROPERTIES OF BLACK HOLES

DISSERTATION SUBMITTED

In Partial fulfillment of the Requirement for the

Award of the Degree of

BACHELOR'S PROGRAMME

IN

PHYSICS

BY

LAKSHMI RAJAN

(Register No-AB20PHY015)

Under the guidance of

Dr. SUNSU KURIAN, ASSISTANT PROFESSOR

Department of physics,

St. Teresa's college (Autonomous), Ernakulam

Kochi, 682011

ST.TERESA'S COLLEGE (AUTONOMOUS)

ERNAKULAM

CERTIFICATE

This is to certify that the project entitled "**THERMODYNAMIC PROPERTIES OF BLACK HOLES"** is a bonafide work done by Lakshmi Rajan of St. Teresa's college Ernakulam, under my supervision at the department of physics, St. Teresa's college Ernakulam for the partial fulfillment of the award of degree of bachelor of science in physics during the academic year 2022-23.The work presented in this dissertation has not been submitted for any other degree in this or any other university.

Supervising guide Head of the department

Dr. Sunsu Kurian, Dr.Priya Parvathy

Assistant professor Assistant professor

Place: Ernakulam

Date:

<u>DECLARATION</u>

I, **LAKSHMI RAJAN(*Reg.No*-AB20PHY015)** final year BSc student ,Department of physics, St. Teresa's college, Ernakulam, do hereby declare that the project work entitled **"THERMODYNAMIC PROPERTIES OF BLACK HOLES"** is an original work carried out under the guidance of Dr.Sunsu Kurian ,Assistant professor, Department of physics ,St. Teresa's college (Autonomous) Ernakulam in Partial fulfillment of the Requirement for the award of the Degree of bachelors programme in Physics.

I further declare that this project is not partially or wholly submitted for any other purpose and the data included in the project is collected from various sources and are true to the best of my knowledge.

Place: Ernakulam

Date:

ACKNOWLEDGEMENT

I bow my head before God Almighty whose grace and blessing enabled me to complete this venture successfully. An opportunity to pay tribute to those who have contributed to the preparation of this project is indeed joyful.

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TABLE OF CONTENTS

ABSTRACT

The term "black hole" is of a very recent origin. It was coined in 1969 by the American scientist John Wheeler, as a graphic description to an idea that dates back to at least two hundred years, to a time when there were two theories about light-the corpuscular theory and the wave theory. We now have a more concise theory in this respect-the wave-particle duality theory of quantum mechanics.

John Michell, in 1783, was the first to provide a breakthrough, through his paper, Philosophical transactions of the royal society of London. In this paper, he pointed out that a star that was sufficiently massive and compact will have such a strong gravitational field that even light cannot escape-any light emitted from the surface of the star will be dragged back by the star's gravitational attraction before it could get very far. Michell suggested that there might be large number of stars just like this. Such entities which form black voids in space came to be known as black holes.

Black hole thermodynamics is a rich subject, straddling both the classical and quantum aspects of gravity. The thermodynamic charges of a black hole such as entropy and temperature, while intrinsically quantum in nature, are related to classical attributes such as horizon area and surface gravity. Indeed, it was considering the classical response of a black hole to infalling matter that led Bardeen, Carter, and Hawking to make the link between black hole variations and laws of thermodynamics. More recently, our understanding of black hole thermodynamics and the interpretation of the various parameters has also been improving.

The outline of this paper is as follows: First, we reviewed the various thermodynamic aspects of non-rotating and uncharged black holes like Reissner- Nordström, BTZ and Bardeen black holes. The findings were then incorporated to investigate, derive and compare thermodynamical parameters of Schwarzschild and Schwarzschild-AdS black holes.

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INTRODUCTION

Thermodynamics is a branch of physics that deals with the transformation of heat into mechanical work. Thermodynamics does not take into account the atomic constitution of matter but it only deals with the macroscopic properties of the system. A system may be defined as a definite quantity of matter (solid, liquid, gas) bounded by some closed surface. The simplest example of a system is a gas contained in a cylinder with a movable piston. Anything outside the system which can exchange energy with it and has a direct bearing on the behavior of the system is known as its surroundings. A system may be separated from its surroundings by a real or an imaginary boundary through which heat or mechanical energy may pass. The existence of a boundary is essential to visualize the system distinctly from the rest of the universe .A thermodynamic system may contain no substance at all, but may consist of radiant energy or electric and magnetic field. The combination of a system and its surrounding is called the universe. The four laws of thermodynamics postulated so far govern the fundamental working of the universe.

During the past 30 years, research in the theory of black holes in general relativity has brought to light strong hints of a very deep and fundamental relationship between gravitation, thermodynamics, and quantum theory. The cornerstone of this relationship is black hole thermodynamics, where it appears that certain laws of black hole mechanics are, in fact, simply the ordinary laws of thermodynamics applied to a system containing a black hole. Indeed, the discovery of the thermodynamic behaviour of black holes — achieved primarily by classical and semi classical analyses — has given rise to most of our present physical insights into the nature of quantum phenomena occurring in strong gravitational fields.

At the purely classical level, black holes in general relativity obey certain laws which bear a remarkable mathematical resemblance to the ordinary laws of thermodynamics.

- Classically, black holes are perfect absorbers but do not emit anything; their physical temperature is absolute zero. However, in quantum theory black holes emit Hawking radiation with a perfect thermal spectrum. This allows a consistent interpretation of the laws of black hole mechanics as physically corresponding to the ordinary laws of thermodynamics.

- The *generalized second law* (GSL) directly links the laws of black hole mechanics to the ordinary laws of thermodynamics.
- The classical laws of black hole mechanics together with the formula for the temperature of Hawking radiation allow one to identify a quantity associated with black holes — namely $A/4$ in general relativity — as playing the mathematical role of entropy. The apparent validity of the GSL provides strong evidence that this quantity truly is the physical entropy of a black hole.

We review the derivation of ordinary laws of thermodynamics applicable to ordinary thermodynamic systems and then extent these derivations to black hole thermodynamics. We then derive the various thermodynamic parameters of Schwarzschild and Schwarzschild Ads black holes. Finally we compare the results to infer the outcome posed to the physical world.

CHAPTER-1

INTRODUCTION

A black hole is a region of space-time where gravity is so strong that no particles or even electromagnetic radiation such as light can escape from it. The story of the black hole begins with Schwarzschild's discovery of the Schwarzschild solution in 1916, soon after Einstein's foundation of the general theory of relativity.

The theory of general relativity predicts that a sufficiently compact mass can deform space time to form a black hole .The boundary of no escape is called the event horizon. Although it has a great effect on the fate and circumstances of an object crossing it, it has no locally detectable features according to general relativity. In many ways, a black hole acts like an ideal black body, as it reflects no light. Moreover, quantum field theory in curved space time predicts that event horizons emit Hawking radiation, with the same spectrum as a black body of a temperature inversely proportional to its mass. This temperature is of the order of billionths of a kelvin for stellar black holes, making it essentially impossible to observe directly.

Objects whose gravitational fields are too strong for light escape were first considered in the 18^{th} century by John Michell and Pierre-Simon Laplace. In 1916, Karl Schwarzschild found the first modern solution of general relativity that would characterize a black hole.

Black holes of stellar mass form when massive stars collapse at the end of their life cycle. After a black hole has formed, it can grow by absorbing mass from its surroundings. Supermassive black holes of millions of solar masses may form by absorbing others stars and merging with other black holes.

The Schwarzschild radius is the boundary of the black hole which is determined by Karl Schwarzschild and it completely depends on the mass of Black hole. If escape velocity is greater than velocity of light, c, nothing can escape its horizon and we have a black hole. Any object with a physical radius smaller than Schwarzschild radius will be a black hole. Anything that crosses the event horizon needs to be travelling at speed greater than velocity of light, c. When a massive star has exhausted the internal thermonuclear fuels in its core at the end of its life, the core becomes unstable and gravitationally collapses inward upon itself and stars outer layers are blown away .The crushing weight of constituent matter falling in from all sides compresses the dying star to a point of zero volume and infinite density called the singularity. This singularity is covered by Event Horizon. Radius of the sphere representing the event horizon is called the Schwarzschild radius,$R_s = \frac{2GM}{c^2}$

NO HAIR THEOREM

The no-hair theorem was originally formulated to describe isolated black holes, but an extended version now describes the more realistic case of a black hole distorted by nearby matter. According to this theorem only three parameters are required to define the most general black hole. They are mass M, charge Q and angular momentum J. Black holes have no hair whereas Star has many hairs(or parameters).

1.1 CLASSES OF BLACK HOLE

Based on no-hair theorem the black holes can be characterized into three

- Static black holes with no charge ,described by Schwarzschild solution
- Black holes with electrical charge described by Reissner Nordstrom solutions
- Rotating black holes described by Kerr solutions

SCHWARZSCHILD BLACK HOLE

Karl Schwarzschild in 1916 gives the First solution of Einstein’s equations of General Relativity. He describes gravitational field in empty space around a non-rotating mass space-time interval in Schwarzschild’s solution. Schwarzschild metric is a spherically symmetric black hole. It is the simplest kind parameterized by a single parameter mass, M. Its line element is defined by:

$$\mathbf{d}\boldsymbol{s}^{\mathbf{2}} = \left(\mathbf{1} - \frac{\boldsymbol{r_s}}{\boldsymbol{r}}\right)\boldsymbol{dt}^{\mathbf{2}} - \left(\mathbf{1} - \frac{\boldsymbol{r_s}}{\boldsymbol{r}}\right)^{-\mathbf{1}}\boldsymbol{dr}^{\mathbf{2}} - \boldsymbol{r}^{\mathbf{2}}(\boldsymbol{d\theta}^{\mathbf{2}} + \boldsymbol{sin}^{\mathbf{2}}\boldsymbol{\theta}\boldsymbol{d}\emptyset^{\mathbf{2}})$$

It exhibits a singularity at the Schwarzschild radius r=2M.This is the surface below which one can no longer escape from the black hole.

REISSNER-NORDSTROM BLACK HOLE

The Reissner-Nordstrom geometry describes the geometry of empty space surrounding a charged black hole. The German engineer, Reissner and the Finnish physicist, Nordstrom independently solved the Einstein-Maxwell field equations for charged spherically symmetric systems ,in 1916 and 1918,respectively.Since most stars ,and thus most black holes are formed from the collapse of stars, have angular momentum ,it is desirable to generalize the spherical, non-rotating Schwarzschild solution to that of rotating source. So the difference from Schwarzschild metric is that this has an additional Coulomb field. The line element for Reissner –Nordstrom black holes is given by:

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2)$$

As the charged black holes in a realistic environment will quickly attract opposite charges from the surroundings and get neutralized, this solution is not of astrophysical interest.

KERR BLACK HOLE

Both the Schwarzschild and Reissner –Nordstrom black holes are spinless. The solution for a rotating black hole was put forward by Kerr in 1963 with an additional 37 parameter, the angular momentum, J. The line element for black holes having mass and angular momentum is given by,

$$\mathrm{d}s^2 = \frac{\Delta}{\rho^2}(dT - hsin^2\theta d\emptyset)^2 - \frac{\rho^2}{\Delta}dR^2 - \rho^2 d0^2 - \frac{sin^2\theta}{\rho^2}[(R^2 + h^2)d\phi - hdT]^2$$

where, h$\cong \frac{J}{M} \approx$ *angular momentum per unit mass,*

$$\triangle = R^2 - 2GMR + h^2$$

$$\rho^2 = R^2 + h^2 cos^2\theta$$

As charged black holes are not considered physically, astrophysical black holes are mainly Kerr.

KERR NEWMAN BLACK HOLE

The Kerr-Newman metric is an asymptotically flat , stationary solution of the Einstein-Maxwell equations in general relativity .It describes the space time geometry in the region surrounding an electrically charged, rotating mass .It generalizes the Kerr metric by taking into account the field energy of an electromagnetic field ,in addition to describing rotation.

Such solutions do not include any electric charges other than that associated with gravitational field and are thus termed as vacuum solutions. Newman combined the RN solution with Kerr solution and generated the space time geometry for charged spinning mass. The metric equation for charged rotating black holes is as same as equation but with Δ defined as

$$\boldsymbol{\Delta = R^2 - 2GMR + h^2 + GQ^2}$$

1.2 BLACK HOLE STRUCTURE

Although black holes come in a variety of masses and sizes, their structures are all alike. A black hole's entire mass is concentrated in an almost infinitely small and dense point called a singularity. This point is surrounded by the event horizon - the distance from the singularity at which its escape velocity exceeds the speed of light. And a rotating black hole is surrounded by the ergosphere, a region in which the black hole drags space itself.

The singularity forms when matter is compressed so tightly that no other force of nature can balance it. In a "normal" star, like the Sun, the inward pull of gravity is balanced by the outward pressure of the nuclear reactions in its core. In the collapsed stars known as white dwarfs or neutron stars, other forces prevent the ultimate collapse.

If there is too much mass in a given volume, though, the object reaches a critical density where nothing can prevent its ultimate collapse to form a black hole.

Because gravity overcomes the other forces of nature, a singularity follows its own bizarre rules of physics. Time and space as we know them are crushed out of existence, and gravity becomes infinitely strong.

As the distance from the singularity increases, the escape velocity decreases. Escape velocity is the speed at which an object must move to get away. For Earth, the escape velocity is around seven miles (11 km) per second. In other words, a spacecraft must go at least that fast to escape Earth's gravitational pull and travel to another planet.

At a certain distance from the singularity, the escape velocity drops to the speed of light (about 186,000 miles/300,000 km per second). This distance is known as the Schwarzschild radius, in honour of Karl Schwarzschild, who first defined it. This radius depends on the mass of the black hole. For a black hole as massive as the Sun, the radius is about two miles (3 km). For every extra solar mass, the radius increases by two miles.

This radius enfolds the singularity in a zone of blackness - in other words, it makes a black hole black. It gives the black hole a visible surface, which is known as the event horizon. This is not a solid surface, though. It is simply the "point of no return" for anything that approaches the black hole. Once any object - from a starship to a particle of light - crosses inside this horizon, it cannot get back out. It is trapped inside the black hole.

Anything that enters the black hole increases its mass. And as the mass goes up, the size of the event horizon gets bigger, too. So if you feed a black hole, it gets fatter!

If the black hole doesn't rotate, then its gravitational influence on its environment is straightforward. If the black hole is spinning, though, then its gravitational effects are more complicated. It actually pulls the fabric of space time along with it - an effect called frame dragging. This area is known as the ergosphere. Seen in cross-section, it is oval-shaped, with the region of influence extending farther into space at the black hole's equator than at its poles.

CHAPTER 2

THERMODYNAMICS

2.1 CLASSICAL THERMODYNAMICS

Thermodynamics is a collection of useful mathematical relations between quantities, every one of which is independently measurable. Although thermodynamics tells us nothing, whatsoever, of the microscopic explanation of macroscopic changes, it is useful because it can be used to quantify many unknowns.

The laws of thermodynamics provide an elegant mathematical expression of some empirically-discovered facts of nature. The principle of energy conservation allows the energy requirements for processes to be calculated. The principle of increasing entropy (and the resulting free-energy minimization) allows predictions to be made of the extent to which those processes may proceed.

The four laws of thermodynamics, even today, serve to be the foundation governing the state of a system. These are:

- Zeroth Law:
  States that if systems A and B are separately in thermal equilibrium with C, then systems A and B are in thermal equilibrium with each other.

- First Law:
  Is consistent with the principle of energy conservation and it states that the energy of an isolated system (one that does not exchange matter or energy with its surroundings) remains constant.

$$\Delta H = q + w_s$$

This equation allows us to calculate the amount of energy transferred to or from any process, simply by calculating the difference in enthalpy before and after. As enthalpy, H, is a state function, its value does not in any way depend on the process

itself or on the imaginary path followed during the process. Enthalpy is a function of temperature and pressure only.
However, the dependence on pressure is small in most cases, and is usually ignored at reasonable pressures.
A process is said to be endothermic when **ΔH > 0**, and exothermic when **ΔH < 0**

- Second Law:

  Is consistent with the principle of increasing entropy. Second law of thermodynamics states that the entropy in an isolated system always increases. Any isolated system spontaneously evolves towards thermal equilibrium—the state of maximum entropy of the system. Thus, the entropy of the universe only increases and never decreases.

- Third Law:

  The entropy of a system at absolute zero temperature either vanishes or becomes independent of the intensive thermodynamic parameters. To bring a system to absolute zero temperature involves an infinite number of processes or steps.

2.2 BLACK HOLE THERMODYNAMICS

It is apparent that energy can flow not just into black holes but also out of them, and they can act as an intermediary in energy exchange processes. Energy extraction is maximally efficient when the horizon area does not change, and processes that increase the area are irreversible, since the area cannot decrease. The analogy with thermodynamic behaviour is striking, with the horizon area playing the role of entropy. This analogy was vigorously pursued as soon as it was recognized at the beginning of the 1970's.

Bekenstein and Hawking showed that the black holes have an entropy which is proportional to the are of the black hole. This was analogous to the second law of thermodynamics.

The entropy of a black hole is given by,

$$S=\frac{\kappa C^3 A}{4G\hbar}$$ Bekenstein Hawking formula

Where A is the area of the event horizon, L_p is the Planck length, G is the Newton's gravity constant, $\hbar$ is the reduced Planck's constant.
Classical aspects of black hole thermodynamics is worth discussing. They are important in their own right, and they form the foundation for quantum black hole thermodynamics. Moreover, it is intriguing to see what can be inferred without invoking quantum theory, and it may teach us something about the deeper origins of gravitation. In proceeding this way we are following more or less the path that was taken historically.

THE FOUR LAWS OF BLACK HOLE MECHANICS

By its very definition, a classical black hole cannot emit anything, so it seemed futile at first to attempt to associate a nonzero temperature with it. On the other hand, there must be some relationship between dM, the change in the mass of a black hole, and dA, the change in its horizon area. According to Penrose process and its charged analogue, when dA = 0, we get

$$dM = \Omega dJ + \Phi dQ,$$

Where J and Q are the angular momentum and charge of the hole and Ω and Φ are the angular velocity and electric potential of the horizon. This expresses changes in the energy of the hole in reversible processes like work done on a thermodynamic system or a change in the number of particles. It is like the First Law of thermodynamics but with the heat flow term dQ = T dS missing.

Zeroth Law

The surface gravity of the horizon of a black hole, κ, is defined locally on the horizon, and it is seen that it's value is always a constant over the horizon of a stationary black hole. This constancy is reminiscent of the Zeroth Law of thermodynamics which states that the temperature is uniform everywhere in a system in thermal equilibrium. Here, κ, plays the role of temperature. This constancy of κ can be traced to the special properties of the horizon of a stationary black hole. Black holes have a well-defined temperature, which is as a matter of fact proportional to the surface gravity:

$$\mathbf{T}=\frac{\hbar}{2\pi}\boldsymbol{\kappa}$$

FIRST LAW

First law for a stationary black hole gives relation between change in mass M, angular momentum J and area A,

$$\mathbf{dM}=\frac{\kappa dA}{8\pi G}+\Omega dJ$$

where Ω is the angular velocity of the event horizon.

For a rotating charged black hole, the first law takes the form,

$$\mathbf{dM}=\frac{\kappa dA}{8\pi G}+\Omega dJ$$

This is analogous to the first law of thermodynamics.

SECOND LAW

The Area theorem of general relativity states that the area of a black hole can never decrease in any process i.e.,

$$\Delta A \geq 0$$

Bekenstein observed that this is analogous to the second law of thermodynamics.

By second law the total entropy of a closed system can never decrease through any process. This law requires black hole to have entropy. If it carried no entropy, falling of mass into a black hole would violate the second law. But this law is not informative in its original form. For example, if an ordinary system falls into a black hole, the ordinary entropy becomes invisible to an exterior observer, so from the observer's point of view, the concept of saying increase in ordinary entropy doesn't provide any insight. Thus, the ordinary second law is transcended.

GENERALIZED SECOND LAW

Including the black hole entropy, gives a more useful law, the generalized second law of thermodynamics, the sum of ordinary entropy outside black holes and the total black hole entropy never decreases and typically increases as a consequence of generic transformations of the black hole. When matter entropy flows into a black hole, the law requires an increase in black hole entropy. During the process of Hawking radiation, the black hole's area decreases, in violation of the area theorem. The generalized second law predicts that the emergent Hawking radiation entropy shall more than compensate for the drop in black hole entropy.

THIRD LAW

It states that it is not possible to form a black hole with vanishing surface gravity. i.e $\kappa = 0$, cannot be achieved. A black hole with T=0, has, $\kappa = 0$. This corresponds to an extreme Kerr black hole with J=M^2

Law	In thermodynamics	For a black hole
Zeroth law	Temperature is uniform in a thermodynamic system in equilibrium	Surface gravity throughout the event horizon is uniform
First law	$dE = TdS +$ work terms	$dm = \frac{\kappa}{8\pi G} dA +$ work terms
Second law	$dA \geq 0$	$dA \geq 0$
Third law	$T = 0$ Cannot be achieved within a finite number of cycles	$\kappa = 0$ Cannot be achieved within a finite number of cycles

CHAPTER 3

THERMODYNAMICS OF SCHWARZSCHILD BLACK HOLE

Karl Schwarzschild in 1916 gave the first solution of Einstein's equations of general relativity. He describes gravitational field in empty space around a non-rotating mass space-time interval in Schwarzschild's solution. Schwarzschild metric is defined for a mass that is spherically symmetric, non-rotating and uncharged.

$$g_{\mu\nu} = \begin{pmatrix} -\left(1-\frac{r_s}{r}\right) & 0 & 0 & 0 \\ 0 & \frac{1}{\left(1-\frac{r_s}{r}\right)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}$$

This provided an exact solution to the Einstein field equations that describes the gravitational field outside a spherical mass, on the assumption that the electric charge of the mass, angular momentum of the mass, and universal cosmological constant are all zero. The solution is a useful approximation for describing slowly rotating astronomical objects such as many stars and planets, including Earth and the Sun. It was found by Karl Schwarzschild in 1916, and around the same time independently by Johannes Droste, who published his more complete and modern-looking discussion four months after Schwarzschild.

According to Birkhoff's theorem, the Schwarzschild metric is the most general spherically symmetric vacuum solution of the Einstein field equations. A Schwarzschild black hole or static black hole is a black hole that has neither electric charge nor angular momentum. A Schwarzschild black hole is described by the Schwarzschild metric, and cannot be distinguished from any other Schwarzschild black hole except by its mass.

The Schwarzschild black hole is characterized by a surrounding spherical boundary, called the event horizon, which is situated at the Schwarzschild radius, often called the radius of a black hole. The boundary is not a physical surface, and a person who fell through the event horizon (before being torn apart by tidal forces), would not notice any physical surface at that position; it is a mathematical surface which is significant in determining the black hole's properties. Any non-rotating and non-charged mass that is smaller than its Schwarzschild radius forms a black hole. The solution of the Einstein field equations is valid for any mass *M*, so in principle (according to general relativity theory) a Schwarzschild black hole of any mass could exist if conditions became sufficiently favorable to allow for its formation.

In the vicinity of a Schwarzschild black hole, space curves so much that even light rays are deflected, and very nearby light can be deflected so much that it travels several times around the black hole.

In Schwarzschild coordinates (t, r, θ, φ),the Schwarzschild metric (or equivalently, the line element for proper time) has the form:

$$ds^2 = -c^2 d\tau^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

Furthermore,

- $d\tau^2$ is positive for timelike curves, in which case τ is the proper time (time measured by a clock moving along the same world line with a test particle)
- c is the speed of light,
- t is, for $r > r_s$,the time coordinate (measured by a clock located infinitely far from the massive body and stationary with respect to it),
- r is, for $r > r_s$,the radial coordinate (measured as the circumference, divided by 2π, of a sphere centered around the massive body),
- φ is the longitude of $\Omega(\text{which is a point on two spheres } S^2)$(also in radians) around the chosen *z*-axis, and
- r_s is the Schwarzschild radius of the massive body, a scale factor which is related to its mass M by $\boldsymbol{r_s = \frac{2GM}{c^2}}$ where G is the gravitational constant

  The Schwarzschild metric has a singularity for r=0 which is an intrinsic curvature singularity. It also seems to have a singularity on the event horizon r=r_s . Depending on the point of view, the metric is therefore defined only on the exterior region $r > r_s$ only on the interior region $r < r_s$ or their disjoint union. However, the metric is actually non-singular across the event horizon. For r>>$r_{s,}$ the Schwarzschild metric is asymptotic to the standard Lorentz metric on Minkowski space. For almost all astrophysical objects, the ratio $\frac{r_s}{R}$ is extremely small. The ratio becomes large only in close proximity to black holes and other ultra-dense objects such as neutron stars.

The radial coordinate turns out to have physical significance as the "proper distance between two events that occur simultaneously relative to the radially moving geodesic clocks, the two events lying on the same radial coordinate line".

The Schwarzschild solution is analogous to a classical Newtonian theory of gravity that corresponds to the gravitational field around a point particle. Even at the surface of the Earth, the corrections to Newtonian gravity are only one part in a billion.

In this chapter, we focus on the thermodynamic parameters of the Schwarzschild black hole by deriving associated thermodynamic parameters like Mass, Radius, Temperature, Entropy and Gibbs free energy. Each relation provides a distinct expression in terms of the horizon radius and length of the black hole.

(i) TEMPERATURE

Line element for spherically symmetric vacuum metric is most familiar in Schwarzschild coordinates:

$$\mathbf{d}\boldsymbol{s}^2 = \left(\mathbf{1} - \frac{\boldsymbol{r_s}}{\boldsymbol{r}}\right)\boldsymbol{dt}^2 - \left(\mathbf{1} - \frac{\boldsymbol{r_s}}{\boldsymbol{r}}\right)^{-1}\boldsymbol{dr}^2 - \boldsymbol{r}^2\left(\boldsymbol{d\theta}^2 + \boldsymbol{sin}^2\boldsymbol{\theta}\boldsymbol{d}\emptyset^2\right) \qquad \text{---(1)}$$

The metric function is given by:

$$\boldsymbol{f}(\mathbf{r}) = \left(\mathbf{1} - \frac{\boldsymbol{r_s}}{\boldsymbol{r}}\right) \qquad \text{-----(2)}$$

The derivative of the metric function w.r.t r is

$$\boldsymbol{f}'(\mathbf{r}) = \left(\frac{\boldsymbol{r_s}}{\boldsymbol{r}^2}\right) \qquad \text{------(3)}$$

Now we compute Bekenstein-Hawking temperature from the above metric easily as follows:

$$\boldsymbol{T_H} = \frac{\boldsymbol{f'}(\mathbf{r})}{\mathbf{4\pi}} \qquad \textbf{-----(4)}$$

At $\mathbf{r}=\boldsymbol{r_+}$, we get the temperature of Schwarzschild black hole as

$$\mathbf{T}=\frac{\mathbf{1}}{\boldsymbol{4\pi r_+}}$$

The Bekenstein–Hawking formula suggests that the microstates are localized on the event horizon.

(ii) RADIUS

The radius of Schwarzschild black hole was accurately derived by Karl Schwarzschild.

Escape velocity for an object of mass M and radius R is given by

$$\mathbf{V_e} = \sqrt{\frac{\boldsymbol{2GM}}{\boldsymbol{R}}} \qquad \textbf{--------------(1)}$$

According to Newton's Second law of motion,

$$\mathbf{F=ma} \qquad \textbf{----------(2)}$$

The force of attraction existing between masses m and M is the force due to gravity and thus, we get,

$$\frac{\boldsymbol{GMm}}{\boldsymbol{r^2}} = \boldsymbol{ma} \qquad \textbf{----------(3)}$$

Therefore,

$$\mathbf{a}=\frac{\boldsymbol{GM}}{\boldsymbol{r^2}} \qquad \textbf{-------------(4)}$$

Or

$$\mathbf{g}=\frac{\boldsymbol{GM}}{\boldsymbol{r_s^2}} \qquad \textbf{----------(5)}$$

Considering PE=KE, we get

$$mgh=\frac{1}{2}mV^2 \quad \text{---------------------(6)}$$

Or

$$V^2 = 2gh \quad \text{------(7)}$$

For a black hole, V=c

Therefore,

$$c^2 = 2gh = \frac{2GM}{r_s^2} \quad \text{----------------(8)}$$

Therefore, Schwarzschild radius is,

$$r_s = \frac{2GM}{c^2}$$

(iii) ENTROPY

Formula for Bekenstein-hawking entropy is given by:

$S=\frac{A}{4l_p^2}$ **,** where A is the area of event horizon and

l_p *is* the Planck length given by

$$l_p = \sqrt{\frac{\hbar G}{c^3}}$$

Or, $l_p^2 = \frac{\hbar G}{c^3}=\frac{hG}{2\pi c^3}$

The surface area is given by,

$$A=4\pi R_{bh}^2$$

$$=4\pi \times \frac{4G^2M^2}{c^4}$$

$$=16\pi \frac{G^2M^2}{c^4} \quad \text{-----(1)}$$

The Bekenstein-Hawking entropy is given by:

$$S=\frac{A}{4l_p^2} \qquad \text{-----(2)}$$

$$=\frac{\frac{16\pi G^2M^2}{c^4}}{\frac{4hG}{2\pi c^3}}$$

$$=\frac{16\pi G^2M^2}{c^4}\times\frac{\pi c^3}{2hG} \qquad \text{------(3)}$$

Therefore, we get the entropy, S, of the black hole as:

$$S=\frac{8\pi^2 GM^2}{hc}$$

The Bekenstein-Hawking entropy or black hole entropy is the amount of entropy that must be assigned to a blackhole in order for it to comply with the laws of thermodynamics as they are interpreted by observers external to that black hole. This is particularly true for the first and second laws.

(iv) MASS

The mass of the black hole can be taken as having an empirical value equal to twice the product of temperature and entropy, i.e.

$$M=2TS \qquad \text{----------(1)}$$

The temperature and entropy was respectively found to be:

$$T=\frac{1}{4\pi r_+} \qquad S=\frac{8\pi^2 GM^2}{hc}$$

Sub. The values in (1), we get:

$$M=2\times\frac{1}{4\pi r_+}\times\frac{8\pi^2 GM^2}{hc}$$

$$=\frac{4\pi GM^2}{r_+hc} \qquad \text{-----------(2)}$$

Therefore, mass, M, is found to be

$$M=\frac{r_+hc}{4\pi G}=\frac{r_+}{2}$$

(v) GIBBS FREE ENERGY

The general equation for Gibb's free energy of a black hole is given by:

$$G=E-TS-\Phi_H Q, \qquad \text{------(1)}$$

Where, E can be approximated to the ADM mass M of the black hole,

T is the temperature,

S, the entropy,

Φ_H, the electrostatic potential at horizon

Q, the charge of the black hole

Hence,

$$E=M \qquad \text{------(2)}$$

For Schwarzschild black hole,

$$TS=\frac{M}{2} \qquad \text{------(3)}$$

$$\Phi_H=\frac{Q}{r_+} \qquad \text{------(4)}$$

Therefore, the Gibbs free energy can be calculated as:

$$G=M-\frac{M}{2}-\frac{Q^2}{r_+} \qquad \text{------(5)}$$

For uncharged black hole, **Q=0**

Hence, we get

$$G=\frac{M}{2}$$

$$=\frac{\frac{r_+}{2}}{2}$$

$$=\frac{r_+}{4} \qquad \text{--------(6)}$$

Therefore, we obtain the Gibbs free energy as:

$$\mathbf{G}=\frac{r_+}{4}$$

CHAPTER 4

THERMODYNAMICS OF SCHWARZSCHILD-AdS BLACK HOLE

Anti-De Sitter spacetime is a crucial ingredient and provides relation between quantum field theory and a higher-dimensional gravity theory. It provides a solution of vacuum Einstein equations in the presence of a negative cosmological constant. The Schwarzschild black hole case offers the possibility to discuss singularities, horizons and boundaries in a simple but non trivial way. It is also interesting to study Schwarzschild black hole in anti-De Sitter space time and its universal covering. Here four dimensional Schwarzschild anti-De Sitter black hole is considered where the mass M of the black hole and the curvature radius of the embedding anti-De Sitter.

In this chapter we consider Schwarzschild-Ads blackhole and derived various thermodynamic quantities.

The solution of the Einstein field equation with the negative cosmological constant is given by:

$$\mathbf{ds^2 = -(1 - \frac{2M(r)}{r} + \frac{r^2}{l^2})\, dt^2 + (1 - \frac{2M(r)}{r} + \frac{r^2}{l^2})^{-1} dt^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)}$$

The Metric function is given by:

$$\boldsymbol{f}\mathbf{(r)= 1 - \frac{2M(r)}{r} + \frac{r^2}{l^2}} \quad \text{--------(1)}$$

(i) MASS:

Mass of Schwarzschild-AdS black hole is obtained by setting *f*(r)=0

$$\mathbf{1 - \frac{2M}{r} + \frac{r^2}{l^2} = 0} \quad \text{---(2)}$$

$$\mathbf{\frac{2M}{r} = 1 + \frac{r^2}{l^2}}$$

$$= \mathbf{\frac{l^2 + r^2}{l^2}}$$

$$M=\frac{r(l^2+r^2)}{2l^2} \qquad \text{----(3)}$$

Sub. $\mathbf{r}=\boldsymbol{r_+}$, we get mass, M, as

$$M=\frac{r_+}{2}\left(1+\frac{r_+^2}{l^2}\right)$$

(ii) TEMPERATURE

The Metric function is given by:

$$f(r)=1-\frac{2M(r)}{r}+\frac{r^2}{l^2} \qquad \text{-----(1)}$$

Derivative of the metric function yields:

$$f'(r)=\frac{2M}{r^2}+\frac{2r}{l^2} \qquad \text{------(2)}$$

Bekenstein-hawking temperature,

$$T_H=\frac{f'(r)}{4\pi}$$

$$T_H=\frac{2Ml^2+2r^3}{4\pi r^2 l^2} \qquad \text{-------(3)}$$

We know,

$$M=\frac{r_+}{2}\left(1+\frac{r_+^2}{l^2}\right) \qquad \text{-----(4)}$$

Substituting, $M=\frac{r(l^2+r^2)}{2l^2}$, we get

$$T_H=\frac{2\left[\frac{r(l^2+r^2)}{2l^2}\right]l^2+2r^3}{4\pi(r^2l^2)} \qquad \text{----------(5)}$$

$$=\frac{rl^2+r^3+2r^3}{4\pi r^2 l^2} \qquad \text{-----------(6)}$$

$$=\frac{rl^2+3r^3}{4\pi r^2 l^2} \qquad \text{---------(7)}$$

$$=\frac{r(l^2+3r^2)}{4\pi r^2 l^2} \qquad \text{---------(8)}$$

$$T_H = \frac{l^2+3r^2}{4\pi r l^2}$$

(iii) RADIUS

Radius is obtained by setting ***f*(r)=0**

$$f(\mathrm{r}) = 1 - \frac{2M(r)}{r} + \frac{r^2}{l^2} \qquad \text{----(1)}$$

$$1 - \frac{2M(r)}{r} + \frac{r^2}{l^2} = 0 \qquad \text{-----(2)}$$

$$\mathrm{r} = \frac{2M + \sqrt{4M^2 - \frac{4}{l^2}}}{\frac{2}{l^2}} \qquad \text{-----(3)}$$

$$= \frac{2M + \sqrt{\frac{4M^2 l^2 - 4}{l^2}}}{\frac{2}{l^2}} \qquad \text{------(4)}$$

$$= \frac{\frac{2Ml + \sqrt{4M^2 l^2 - 4}}{l}}{\frac{2}{l^2}} \qquad \text{-----(5)}$$

$$= \frac{\left(2Ml + \sqrt{4M^2 l^2 - 4}\right) l}{2} \qquad \text{------(6)}$$

$$= \frac{(2Ml + 2\sqrt{(ml)^2 - 1})\, l}{2} \qquad \text{-------(7)}$$

$$= \frac{2Ml^2}{2} + \frac{2l\sqrt{(ml)^2 - 1}}{2} \qquad \text{-------(8)}$$

$$r_+ = \mathrm{m}l^2 + l\sqrt{(ml)^2 - 1}$$

(iv) ENTROPY:

Entropy of the black hole is proportional to the area of its event horizon. Detailed analysis by Hawking showed that entropy is given by:

S=$\frac{A}{4l_p^2}$, where A is the area of event horizon and

l_p *is* the Planck length ---(1)

The surface area of the black hole is given by

$$\mathbf{A=4\pi r_+^2} \quad \textbf{---(2)}$$

Considering Schwarzschild-AdS black hole to be of minimum length

i.e. , $\boldsymbol{l_p = 1}$

Therefore,

$$\mathbf{S=\frac{4\pi r_+^2}{4*1}} \quad \textbf{---(3)}$$

Hence, entropy

$$\mathbf{S= \pi r_+^2}$$

<u>(v) GIBBS FREE ENERGY:</u>

The general equation for Gibb's free energy of a black hole is given by:

$$\mathbf{G=E\text{-}TS\text{-}\Phi_H Q,} \quad \textbf{------(1)}$$

Where, E can be approximated to the ADM mass M of the black hole,

T is the temperature,

S, the entropy,

Φ_H, the electrostatic potential at horizon

Q, the charge of the black hole

Hence,

$$\mathbf{E=M} \quad \textbf{------(2)}$$

For Schwarzschild black hole,

$$\mathbf{TS=\frac{M}{2}} \quad \textbf{------(3)}$$

$$\Phi_H = \frac{Q}{r_+} \qquad \text{------(4)}$$

Therefore, the Gibbs free energy can be calculated as:

$$G = M - \frac{M}{2} - \frac{Q^2}{r_+} \qquad \text{------(5)}$$

For uncharged black hole, **Q=0** $\quad G = \frac{M}{2} \qquad \text{------(6)}$

$$= \frac{\frac{r_+}{2}\left(1 + \frac{r_+^2}{l^2}\right)}{2}$$

$$G = \frac{r_+}{4}\left(1 + \frac{r_+^2}{l^2}\right)$$

CHAPTER 5

COMPARISON AND INFERENCE

PARAMETERS	SCHWARZCHILD	SCHWARZCHILD ADS
TEMPERATURE	$T=\frac{1}{4\pi r_+}$	$T_H = \frac{l^2 + 3r^2}{4\pi r l^2}$
RADIUS	$r_s = \frac{2GM}{c^2}$	$r_+ = ml^2 + l\sqrt{(ml)^2 - 1}$
ENTROPY	$S = \frac{8\pi^2 GM^2}{hc}$	$S = \pi r_+^2$
MASS	$M=\frac{r_+ hc}{4\pi G} = \frac{r_+}{2}$	$M=\frac{r_+}{2}\left(1 + \frac{r_+^2}{l^2}\right)$
GIBBS FREE ENERGY	$G=\frac{r_+}{4}$	$G = \frac{r_+}{4}\left(1 + \frac{r_+^2}{l^2}\right)$

The Einstein equations with a negative cosmological constant admit black hole solutions which are asymptotic to anti-de Sitter space. Like black holes in asymptotically flat space, these solutions have thermodynamic properties including a characteristic temperature and an intrinsic entropy equal to one quarter of the area of the event horizon in Planck units. There are however some important differences from the asymptotically flat case. A black hole in anti-de Sitter space has a minimum temperature which occurs when its size is of the order of the characteristic radius of the anti-de Sitter space. For larger black holes the red-shifted temperature measured at infinity is greater. This means that such black holes have positive specific heat and can be in stable equilibrium with thermal radiation at a fixed temperature.

All parameters of the Schwarzschild-AdS black holes was found to depend on the length and radius of AdS space Unlike, Schwarzschild black hole, whose thermodynamic properties were found to depend only on the horizon radius.

Both black holes have was found to have positive values of Gibbs free energy which indicated the phase transitions.

All parameters of Schwarzschild-AdS black holes tend to minimum if the length is considered to be a minimum.

CONCLUSION AND FUTURE SCOPE

In this project we focused on how the laws of thermodynamics changes with the existence of black hole event horizon.We study the thermodynamics of Schwarzschild black hole and derive the thermodynamic properties of Schwarzschild black holes in ADS space. Different thermodynamics properties such as Entropy, temperature, Gibbs free energy etc. are calculated and a comparative study of both systems is carried out.

As a further investigation, one can also consider the micro canonical ensemble. One can avoid the problem that arises in asymptotically flat space of having to put the system in a box with unphysical perfectly reflecting walls because the gravitational potential of anti-de Sitter space acts as a box of finite volume. It also implies that the canonical ensemble exists for asymptotically anti-de Sitter space, unlike the case for asymptotically flat space.

Later, the Hawking-Page phase transition (i.e., a phase transition between the thermal anti-de Sitter (AdS) space and a black hole) can be studied in Schwarzschild-AdS black holes. Unlike Schwarzschild black holes, Schwarzschild-AdS black holes can be thermally stable since the AdS boundary acts as a reflecting wall for the Hawking radiation.

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