TM211490TR

Reg. No :...............................

Name :..................................

M. Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2021

[2021 Admissions Regular and 2020 Admissions Improvement & Supplementary]

SEMESTER I - CORE COURSE (MATHEMATICS)

MT1C02TM20 - BASIC TOPOLOGY

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Define (i) Sierpinski Topology (ii) Order Topology
2. Prove that a subset of a topological space is open if and only if it is a neighbourhood of each of its points.
3. Prove or disprove that in a metric space a closed ball is the closure of the open ball with the same centre and radius.
4. Prove that constant function is always continuous.
5. Prove that in a co-finite space any finite subset is closed.
6. Define homeomorphism. Give three equivalent conditions for $f : X \to Y$ to be a homeomorphism.
7. Prove that every continuous image of a compact space is compact.
8. Prove that regularity is a hereditary property. Prove that every continuous image of a compact space is compact.
9. Prove that every path connected space is connected.
10. Is every quotient map open?Justify.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Let (X,d) be a metric space then prove that
    (i) The empty set and the full set X are open sets.
    (ii) Arbitrary union of open sets is again open.
    (iii) Finite intersection of open sets is again open.
    (iv) Given distinct points x,y $\in$ X, $\exists$ open sets U and V such that x $\in$ U and y $\in$ V and U $\cap$ V= $\emptyset$.
12. Define hereditary property with example. Prove that second countability is a hereditary property.
13. Define a dense subset in a topological space (X,T).If Bis any base for (X,T), prove that a subset D of X is dense in X if and only if D intersects every non-empty member of B.
14. Define interior of a set. Let X be a space and A be a subset of X. Then prove that int(A) is the union of all open sets contained in A. Also prove it is the largest open subset of X contained in A.
15. Prove that a topological space X is regular if and only if for any x ∈ X and any open set G containing x, there exists an open set H containing x such that H^{-}.
16. Show that the topological product of of any finite number of connected space is connected.
17. Show that every closed surjective map is a quotient map. And hence show that every quotient space of a discrete space is discrete.
18. Prove that every quotient space of a locally connected space is locally connected.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. Define a sub-base for a topology. Give an example of a sub-base for the set of real numbers with usual topology. Also prove that if X is a set, T a topology on X and S, a family of subsets of X, then S is a sub-base for T if and only if S generates T.
20. State and Prove the equivalent conditions for a function being continuous.
21. Prove that every closed and bounded interval is compact.
22. (a) Define T4 and prove that all metric spaces are T4. (b) Show that every regular, Lindeloff space is normal.