TM211500TR **Reg. No :...............................**

Name :.................................

M. Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2021

[2021 Admissions Regular and 2020 Admissions Improvement & Supplementary]

SEMESTER I - CORE COURSE (MATHEMATICS)

MT1C03TM20 - REAL ANALYSIS

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. If f is continuous on [a,b] and if f' exist and is bouded in the interior say $|f'(x)| \leq A \; \forall \, x \in (a,b)$ then prove that f is of bounded variation on [a,b].
2. Prove or disprove, "f(x)=x on [0,1] is of bounded variation"
3. If $U(P,f,\propto) - L(P,f,\propto) < \epsilon$ holds for some P and some ε then prove that the same equation holds for every refinement of P.
4. If $f \in R(\propto)$, then prove that $|\int_a^b f \, d\propto| \leq \int_a^b |f| \, d\propto$
5. Prove or Disprove, "Limit of the Integral need not be equal to the integral of the limit even if both are finite."
6. Give an example of an everywhere discontinuous limit function, which is not Riemann-Integrable.
7. Prove or Disprove, " The limit processes cannot in general be interchanged without affecting the result"
8. State and prove any two properties of logarithmic function.
9. State and prove any two properties of exponential function.
10. If E(z) = $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ prove that E(z+w) = E(z) E(w).

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. State and prove the sufficient condition for bounded variation.
12. Prove or Disprove, "Boundedness of f^' is not necessary for f to be of Bounded variation"
13. Suppose f is bounded on [a,b], f has only finitely many points of discontinuity on [a,b] and $\propto$ is continuous at every point at which f is discontinuous. Then prove that $f \in R(\propto)$
14. If $f \in R(\propto)$ and $g \in R(\propto)$ on [a,b] then prove that

    a. $fg \in R(\propto)$ and

    b. |f| $\in R(\propto)$ and $|\int_a^b \mathrm{f} \, d\alpha| \leq \int_a^b |\mathrm{f} \, d\alpha|$

15. State and prove Weistrass-M-Test for Uniform Convergence.
16. Let {fn}, {gn} which converge uniformly on some set E, prove that {fngn} doesnot converge uniformly on E.
17. State and prove Abel's Theorem.

18. If $\{f_n\}$ is a point wise bounded sequence of complex functions on a countable set E, then prove that $\{f_n\}$ has a subsequence $\{f_{n_k}\}$ such that $\{f_{n_k}(x)\}$ converges for every x $\in$ E.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. Let f be of bounded variation on [a,b]. If x $\in$ (a,b], let $V(x)=V_f(a,x)$ and put V(a) = 0. Then prove that every point of continuity of f is also a point of continuity of V and the converse is also true.
20. State and prove the five properties of the Integrals.
21. State and prove the relationship between Uniform convergence and Integration.
22. Suppose $a_0, a_1, \dots, a_n$ are complex numbers, $n \geq 1, a_n \neq 0, P(z) = \sum_{n=0}^{\infty} a_k z^k$. Then show that P(z) = 0 for some complex number z.