TM211480TR **Reg. No :..............................**

Name :................................

M. Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2021

[2021 Admissions Regular and 2020 Admissions Improvement & Supplementary]

SEMESTER I - CORE COURSE (MATHEMATICS)

MT1C01TM20 - LINEAR ALGEBRA

Time : 3 Hours **Maximum Weight : 30**

Part A

I. Answer any Eight questions. Each question carries 1 weight **(8x1=8)**

1. Let α be a vector in F^n. Find the coordinate matrix of α relative to the standard basis of F^n.
2. Verify whether ($\mathbb{R}^n$, $\oplus, \cdot$) with the two operations defined by $\alpha \oplus \beta = \alpha - \beta$ and c .α = - c α is a vector space.
3. If W is a k-dimensional subspace of an n-dimensional vector space V, then, show that W is the intersection of (n – k) hyperspaces in V.
4. Define dual space of V and show that dim V* = dim V.
5. Prove that a linear transformation T is non singular if and only if , T is one-one.
6. For a 2x2 matrix A over a field prove that det(I+A)=1+detA if and only if trace(A) = 0.
7. Let D be a 2-linear function with the property that D(A) = 0 for all 2 X 2 matrices A over a commutative ring with identity, K, having equal rows. Then show that D is alternating.
8. Determine a 3×3 matrix for which the minimal polynomial is x^2.
9. Define a projection of a vector space V. Suppose E is a projection and let R be the range of E and N be the null space of E. Then show that the vector β is in the range R if and only if Eβ = β.
10. Prove that every matrix A such that $A^2 = A$ is similar to a diagonal matrix.

Part B

II. Answer any Six questions. Each question carries 2 weight **(6x2=12)**

11. Let W_1 and W_2 be subspaces of a vector space V such that $W_1 + W_2 = V$ and $W_1 \cap W_2 = \{0\}$. Prove that for each vector α in V there are unique vectors α_1 in W_1 and α_2 in W_2 such that $\alpha = \alpha_1 + \alpha_2$.
12. Let $\mathfrak{B} = \{\alpha_1, \alpha_2, \alpha_3\}$ be the ordered basis for R^3 consisting of $\alpha_1 = (1,0,-1)$, $\alpha_2 = (1,1,1)$ $\alpha_3 = (1,0,0)$. What are the coordinates of the vector (a, b, c) in the ordered basis $\mathfrak{B}$?
13. If V and W are finite dimensional vector space over a field F, prove that V and W are isomorphic if and only if dim V = dim W.
14. Let T be a linear transformation from V into W where V and W are finite dimensional vector spaces over the field F. Show that rank (T^t) = rank (T).
15. Prove that an n × n matrix A over a commutative ring with identity, K, is invertible if and only if det A is invertible in K.
16. Show that the determinant function on 2 X 2 matrices A over K, a commutative ring with identity, is alternating and 2-linear as a function of the columns of A.

17. Let T be a linear operator on V, a finite dimensional vector space and *c* be a scalar. Then prove that the following statements are equivalent.

   1. c is characteristic value of T.
   2. The operator (T – c I) is singular.
   3. $|T - cI|$ = 0.

18. Find the minimal polynomial for T where T is a linear operator on R^3 which is represented in the standard ordered basis by the matrix $\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$.

Part C

III. Answer any Two questions. Each question carries 5 weight **(2x5=10)**

19. Let m and n be positive integers and let F be a field. Suppose W is a subspace of F^n and $\dim W \leq m.$ Prove that there is precisely one m x n row reduced echelon matrix over F which has W as its row space.

20. (a) V and W be finite dimensional vector spaces over the field F and let $\{\alpha_1, \alpha_2 \dots \dots \alpha_n\}$ be an ordered basis for V and let ,$\beta_1, \beta_2,$. . ., β_n be any vectors in W. Then show that there is precisely one linear transformation $T: V \rightarrow W \text{ such that } T\alpha_j = \beta_j; j = 1,2, \dots n$.

    (b) Let F be a field. $f: F^2 \rightarrow F$ is defined by $f(x_1, x_2) = ax_1 + bx_2.$ Define $T: F^2 \rightarrow F^2$ as $T(x_1, x_2) = (-x_2, x_1)$. Compute $T^t f$.

21. State and prove the properties of determinants.

22. (a) Suppose T be a linear operator on the n dimensional vector space V and suppose that T has n distinct characteristic values. Prove that T is diagonalizable.

    (b) Let V be a finite dimensional vector space over the filed F and let T be a linear operator on V. Show that T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F.